Voluntary Disclosure and the Duty to Disclose

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Abstract

This paper evaluates firms’ disclosure decisions when firms have a duty to disclose the material information in their possession. When firms are caught violating that duty, they are liable for damages. The damage payments in the model reflect many features of damage payments in 10b-5 litigation. Predictions are developed regarding the dependence of each of: a firm’s equilibrium voluntary disclosure policy, its equilibrium investment decision, and its equilibrium market price on various parameters of the model, including: the frequency the firm receives material information, the prevailing norm for what constitutes material information, the expected size of the damage payments for withholding material information, the expected fraction of the firm’s shareholders who purchase the firm’s shares while the material information is withheld, and the probability the firm is detected having withheld material information.

The paper also makes predictions about the consequences of altering the prevailing method of assigning damage payments for withholding material information.

1 Introduction

The existing models of voluntary disclosure assume that all disclosures under consideration are voluntary: in these models, at the firm’s (i.e., its managers’) sole discretion, the firm can withhold or disclose whatever private information it receives. Withholding information may result in a higher price for the firm than disclosing the information, but there is no overt penalty for failure to disclose value-relevant information. This is clearly not the case in practice. For example, the famous SEC Rule 10b-5 concerning the Employment of Manipulative and Deceptive Practices states that:

"It shall be unlawful for any person, directly or indirectly, by the use of any means or instrumentality of interstate commerce, or of the mails or of any facility of any national securities exchange, to....make any untrue statement of a material fact or to omit to state a material fact necessary in order to make the statements made, in the light of the circumstances under which they were made, not misleading..in connection with the purchase or sale of any security."

Other SEC regulations impose similar affirmative duties on firms to make disclosures. For example, SEC Rule 12b-20 stipulates that:

"In addition to the information expressly required to be included in a statement or report, there shall be added such further material information, if any, as may be necessary to make the required statements, in the light of the circumstances under which they are made not misleading."
Firms found in violation of these requirements can be subject to civil or, in some cases, criminal penalties by the SEC, as well as the subject of private litigation by wronged investors.

Notwithstanding the affirmative duty firms have to make disclosures to be in conformity with these and related SEC rules (as well as other GAAP disclosure requirements), it would be incorrect to postulate that firms do not exercise discretion in deciding whether to disclose their private material information ("MI"). Just as motor vehicle drivers evaluate the benefits and costs of obeying mandatory speeding limits - and bear the penalties if they get caught exceeding the posted limits, corporate managers evaluate the benefits and costs of complying with the above cited (and related) disclosure requirements - and bear the penalties if they choose to withhold MI from the marketplace. The plethora of SEC enforcement actions and private civil litigation involving firms accused of withholding MI is testament to firms exercising their discretion in deciding whether to obey mandatory disclosure requirements.\(^1\)

This paper develops a model that evaluates expected value-maximizing firms' disclosure decisions when the firms have a duty to disclose MI in their possession. When firms are caught violating that duty, they are liable for damages. The damage payments in the model reflect many features of damage payments in 10b-5 litigation. The damage payments are the product of: (1) the difference between the "artificially high" market price of the firm that prevailed during the time the firm withheld MI and the market price that would have prevailed during this time had the firm disclosed the MI; (2) the number of shares investors purchase at the "artificially high" price; and (3) a damages "multiplier."\(^2\)

The model reflects several other features of the institutional environment in which voluntary disclosure decisions take place too, including: 1. the materiality of the withheld information is determined by assessing how much the firm's stock price would have changed had the withheld information been disclosed; 2. the firm's failure to divulge MI is only occasionally detected by some "fact finder"; 3. legal fees arising from the assessment of damages have both fixed and variable cost components; 4. there is uncertainty at the time the firm decides whether to withhold MI from investors regarding the number of shares that investors will purchase at the "artificially high" price that will prevail if the firm elects to withhold its unfavorable information from the capital markets; 5. damages are paid by the firm after the fact finder has determined that MI was withheld, and hence the damages are effectively paid by the shareholders of record after the fact finder's determination.

Another feature of the model is that it distinguishes between information that arrives at a firm and information that the firm's senior management is aware has arrived at the firm. This distinction affects a firm's voluntary disclosure decisions because, naturally, its senior managers cannot decide whether to disclose information that the managers do not know the firm possesses. We posit that a firm is liable for failing to disclose MI known somewhere inside the firm even if the firm's senior managers are unaware of the information, since courts may reasonably assert that its senior executives "either knew or should have known" the information. We speculate that this may be an avenue for a firm's organizational design to

\(^1\) Sunder [1997] emphasizes the point that the distinction between "mandatory" and "voluntary" actions is often artificial (see pages nine and ten of Sunder [1997]).

\(^2\) The size of the damages multiplier is taken as exogenous in this paper, but is affected in practice by some combination of the goals the courts, arbitrators, etc., adjudicating the cases (e.g., reimbursing "wronged" shareholders; penalizing firms for having withheld the material information, etc.) and the skills of the parties' legal representatives.
affect both its voluntary disclosure decisions and its liability for withholding MI from capital markets, as some organizational forms would seem to lead to more upward internal information flows that others. For example, in centralized, hierarchical firms information naturally would seem to percolate up the chain of command more than it would in decentralized, divisional firms.

We derive many empirical implications from our analysis. We show how a firm's senior managers' propensity to withhold MI they receive is affected by each of: the fraction of the firm's shares expected to be purchased at the "artificially high" price when MI is withheld, the size of the damages multiplier, the probability the senior managers of a firm are aware of MI arriving at their firm, the probability a fact finder will determine that a firm has withheld MI, the size of the litigation costs the firm incurs, and the determination of what is considered to be MI. We also show how changes in the preceding variables influences each of: the firm's preferred (endogenous) investment levels, the firm's equilibrium "ex ante" (pre-information receipt) expected market values, and the firm's equilibrium "ex post" (post-information receipt) expected market values.

As far as we are aware, these results are all new to the literature on voluntary disclosures.

In the remainder of this introduction, we informally preview one subset of our results - those related to the economic effects of a change in the materiality threshold ("MT"). A MT determines the limit on how bad the private information a firm withholds from the capital markets can be without subjecting the firm to liability for failing to disclose information in the event the firm's withholding is subsequently discovered.

We begin by showing that, regardless of the level of the MT, a firm's equilibrium disclosure policy is described by a "cutoff" policy where the firm discloses the information it receives if the information is above the cutoff and it withholds the information it receives if the information is below the cutoff. We then show that this cutoff is not monotonic in the MT. That is, whether a firm is more likely or less likely to disclose the private information in its possession is not monotonic in the level of the MT. When the MT is sufficiently low, we show that an expected value-maximizing firm is less likely to disclose its private information if the MT is raised somewhat, but if the MT is sufficiently high, an expected value-maximizing firm is more likely to disclose its private information if the MT is further increased. What explains the difference? When the MT is sufficiently high, firms have considerable discretion in deciding what they disclose or withhold without fear of facing legal reprisals for failing to disclose their private information. Investors become progressively more concerned as high materiality thresholds are increased that a firm's lack of disclosure is attributable to the firm having received and withheld unfavorable information rather than being attributable to the firm not having received information, and so they discount the price of the firm more when the firm makes no disclosure. In equilibrium firms respond to investors' increased discounting by disclosing more of the information they receive. In contrast, when the MT is initially very low, and hence a firm runs a considerable risk of being subject to legal penalties in the event it fails to disclose most of the private information in its possession, investors believe that even if the MT is increased somewhat, a firm will not greatly increase its propensity to withhold the adverse private information it receives and bear the risk of being found liable for withholding MI, and so investors don't discount the firm's price more in the event the firm withholds

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3Because of concerns about space, we leave the discussion of the economic effects of changing the other parameters of the model to the body of the text.
information. In equilibrium, firms respond to the combination of investors’ lack of increased discounting and the increased license to withhold information (resulting from the increase in the MT) by withholding somewhat more information than they would have with a lower MT.

We also show, when investment is endogenous, that the effect of changing the MT on the firm’s equilibrium cutoff is in the opposite direction from the effect of the change in the MT on both the firm’s equilibrium investment level and the equilibrium expected value of the firm. (This inverse relationship prevails for changes in all the other parameters of the model too.) This is explained by the fact that the equilibrium cutoff also constitutes the equilibrium price the firm receives in the event the firm makes no disclosure, and so a change in any parameter that increases (resp., reduces) the equilibrium cutoff increases (resp., reduces) the equilibrium price when the firm makes no disclosure. Parameters that result in an increase in the firm’s "no disclosure" price reduce the firm’s incentives to invest in productive activities that increase the firm’s value for two related reasons: first, because the higher the "no disclosure" price, the less often the firm discloses the information it receives, and second, the higher the "no disclosure" price, the less the firm needs to work to "show" that it has earned the value investors assign to it in the capital markets. These two mutually reinforcing effects result in an inverse relationship both between the firm’s equilibrium cutoff and its equilibrium investment level and between the firm’s equilibrium cutoff and its equilibrium expected market value.

In addition to investigating the effects of changing the MT, as well as other parameters of the model, on the firm’s equilibrium pricing and investment choices, we also consider how the predictions of the model would change were the burden of the damage payments shifted away from the firm (really, the firm’s shareholders) after the fact finder discovered the firm withheld MI and shifted toward the firm’s shareholders at the time the firm made the decision to withhold MI. The reason for evaluating such a shift in the assignment of damages is this: there is widespread criticism of the prevailing method of assigning damages in 10b-5 litigation, basically because the shareholders responsible for paying the damages did not commit the tort (withholding MI) that gave rise to the damage payments. It would seem preferable to make the damage payments the responsibility of those who either committed, or benefited from, the tort. Those who committed the tort are, of course, the senior managers of the firm, and those who benefited from the tort (besides the senior managers) are the shareholders of record when the tort took place. Since - in our model - managers have no preferences independent of shareholders (though the model could be extended to incorporate agency considerations that derive from divergent preferences of managers and shareholders), the preferred reassignment of damage payments entails having the shareholders of record at the time the disclosure decision took place make the damage payments.

It has - to our knowledge - never been made clear in the literature how reassigning damages in this way would alter the firm’s incentives to make disclosures, presuming the practical problems associated with this reassignment of this liability could be surmounted. We show that, depending on the values of the parameters of the model, sometimes the equilibrium disclosure policies that result from the reassignment of damages are exactly the same as the disclosure policies that prevail with the conventional assignment of damages, but other times they are radically different. Under the conventional assignment of liability for

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4See, e.g., Alexander [1996], Spindler [2008].
damages, equilibrium disclosure policies are always contiguous, in the sense that if a firm received private information which, if disclosed, would result in the firm’s value being one of the values $v, v', v'', v'''$ where $v < v' < v'' < v'''$, and if it is optimal for the firm to withhold (resp., disclose) information that gave rise to the firm’s value being $v'$ (resp., $v''$), then it is also optimal for the firm to withhold $v$ (resp., disclose $v'''$). But, with the reassignment of damages, we show that, naturally - without adopting any unusual parameter specifications - the equilibrium disclosure policy sometimes has large "gaps" in it, in so far as it there exist $v, v', v'', v'''$ (with the same meaning and the same ordering $v < v' < v'' < v'''$ as above) such that the firm: does not disclose the information associated with $v$, does disclose the information associated with $v''$, and does disclose the information associated with $v'''$.

In terms of related literature: as is evident from these introductory remarks, this paper is premised on the idea that firms sometimes receive private information pertinent to their accurate pricing, but which they would prefer not to disclose. For this premise to be valid, a setting must be considered where Grossman’s [1981] and Milgrom’s [1981] "unravelling" result does not apply. Dye [1985] and Jung and Kwon [1988] identified a natural setting in which firms are inclined to disclose their information selectively, which also has been adopted by other researchers in their examination of firms’ voluntary disclosure policies (see, e.g., King and Wallin [1991], Pae [2005], Shavell [1994]), arising as a consequence of the firms probabilistically receiving private information relevant to their valuation. We make use of this natural setting here. This paper can be also be viewed as responding to the observation of Heitzman, Wasley and Zimmerman [2010] who note that many "voluntary" disclosures are not really voluntary, in so far as firms often have an affirmative duty to disclose the MI in their possession. Related, the paper studies how changes in various parameters of the model change optimal firm behavior and expected firm value, with special emphasis given to the effects of changes in materiality thresholds. While conceptual studies of materiality thresholds were at one time quite prevalent in the accounting literature (see, e.g., Patillo [1976], Ward [1976]), of late materiality has been principally the focus of empirical studies (see, e.g., Messier, Martinov-Bennie, and Aasmund Eilifsen [2005] and the previously mentioned paper by Heitzman, Wasley and Zimmerman [2010]).

Some parts of the present paper document when and why firms might be inclined to disclose bad news. This question has fascinated both analytical and empirical accounting researchers. On the empirical side, see, e.g., Skinner [1994],[1997], and on the analytical side, see, e.g., Wagenhofer [1990], Suijs [2005], [2007]. This paper is part of the vast literature in law and economics that assesses damage rules related to 10b-5 litigation. Posner [2007] discusses some of the issues raised in this literature, as do, for example, Alexander [1996] and Booth [2009].

The paper proceeds as follows. The next section, section 2, introduces the base model. In the base model, investment is taken as exogenous, and damages related to the withholding of MI are assessed against the firm when a fact finder discovers that the firm withheld MI from investors. In section 3, the equilibrium of the base model is defined. Section 4 analyzes the equilibrium of the base model. Section 5 extends the base model to include endogenous investment. Section 6 considers the consequences of reassigning damage payments to the shareholders of record circa the time the firm decided whether to withhold or disclose the information in its possession. Section 7 contains conclusions. The Appendix contains the proofs of most results not proven in the text or accompanying footnotes.
2 Introduction to the Base Model

The time line of the base model runs as follows. The model starts at "time 0." Between time 0 and some later point in time, "time 1," a firm with probability \( p \) privately receives some value-relevant information (which we take to be the realization \( x \) of some random variable \( \tilde{x} \) with cumulative distribution function ("cdf") \( F(x) \) and support \([x, \tilde{x}]\)). With probability \( 1 - p \), the firm receives no information by time 1. If the firm learns the realization of \( \tilde{x} \), it may disclose that information or disclose nothing by, say, "time 2" (a time greater than time 1). (In the following, time 2 is sometimes alternatively referred to as the "disclosure date." ) Disclosing or withholding the information is its only options. It cannot, for example, "partially" disclose the information it receives, and it cannot distort the information it receives before disclosing it.\(^5\) If the firm does not learn information, it necessarily makes no disclosure.

If the firm discloses \( x \), there is some strictly increasing function \( v(x) \) such that the per share price of the firm at time 2 is \( v(x) \).\(^6\)

If the firm does not disclose anything by time 2, then the per share price of the firm at time 2, \( P^{nd} \), is the expected value of \( v(\tilde{x}) \) – suitably adjusted for any estimated damage payments the firm is liable for in the event it is subsequently determined that the firm withheld MI in its possession from investors at time 2 (as described further below). So, risk neutral pricing prevails. In the event the firm makes no disclosure, investors are also assumed, as of time 2, to have no other sources of information about \( \tilde{x}'s \) realization, so the "suitably adjusted" expected value of \( v(\tilde{x}) \) is based entirely on whatever information investors can infer about \( \tilde{x}'s \) distribution from the firm’s nondisclosure. Investors are also assumed to be homogeneously informed, so this expected value is common to all investors. In the event the firm receives information but does not disclose it by time 2, it never discloses it (at least within the time horizon considered in the model).

We adopt the simplification that no information arrives between times 2 and "time 3" ("time 3" is a time after time 2, described next), so if the firm made no disclosure by time 2, all investors who buy shares of the firm between times 2 and 3 all purchase those shares at the same (per share) price \( P^{nd} \). The realization of the random variable \( \tilde{f} \) with prior mean \( \bar{f} \) represents the fraction of the firm’s shares purchased by investors between times 2 and 3.

If the firm withholds information, say \( x \), in its possession, there is some fact finder - perhaps an investor, auditor, reporter, market maker, etc. - who, at time 3 probabilistically discovers that the firm did not disclose that it knew \( x \) by time 2. (Time 3 in the following is sometimes alternatively referred to as the "discovery date.") The probability the fact finder may detect the firm’s previous withholding of the information \( x \) is given by the realization of the random variable \( \tilde{q} \) with mean \( \bar{q} \). In the model, while the fact finder may fail to discover that the firm withheld MI, the fact finder is assumed not to err by claiming that the firm withheld MI when that is not the case.

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\(^5\) The presumption here that the firm would not partially disclose its information need not be assumed: it can be derived as a feature of any equilibrium (because any incomplete disclosure must be interpreted by investors as skeptically as possible in equilibrium, so a firm that disclosed that it received information by time 1 would be obliged to disclose the exact value of its information to avoid unfavorable pricing by skeptical investors; this is a manifestation of the "unravelling" result of Grossman [1981] and Milgrom [1981]).

\(^6\) Also, the presumption that the firm cannot distort the information it receives is based on the implicit premise that antifraud statutes are sufficiently great and sufficiently well enforced so as to deter distorted disclosures.

Consistent with the preceding description, unless something explicitly to the contrary is stated, all firm values should be taken to be amounts per share.
If the firm receives information but fails to disclose it, the firm may be liable for damages, if the information withheld is deemed material. The withheld information $x$ is deemed *immaterial* if the difference between the price the firm’s shares trade at between times 2 and 3 when the firm makes no disclosure, $P^{nd}$, and the price $v(x)$ those shares would have traded at had $x$ been disclosed, is less than the MT, denoted by $\Delta_m$, i.e.,

$$P^{nd} - v(x) < \Delta_m;$$

otherwise, the firm is considered to have withheld MI.

When the firm is deemed to have withheld MI, the firm is obliged to pay all of the investors who purchased the firm’s shares at the price $P^{nd}$ between times 2 and 3 the amount $\beta(P^{nd} - v(x))$ per share, where $\beta$ is a positive "damages multiplier" determined by one or some combination of a judge, negotiations between plaintiffs (investors who purchased the firm’s shares between times 2 and 3) and defendant (the nondisclosing firm), arbitration, etc.\(^7\) Since in practice, the damages multiplier $\beta$ will be uncertain at the time the firm makes its disclosure decision at time 2, we represent $\beta$ as the realization of some random variable $\tilde{\beta}$ with mean $\bar{\beta}$. In practice, $\tilde{\beta} \leq 1$ virtually always holds,\(^8\) and so, unless otherwise stated, we shall adopt that as a maintained assumption. The total damage payments the firm is liable for, if it turns out that fraction $f$ of the firm’s shares are purchased between times 2 and 3, reduces the per share value of the firm by $f \beta(P^{nd} - v(x))$.

In addition, as was discussed in the Introduction, the firm may have to incur legal costs to have itself represented in the proceedings related to the assessment of damages. These legal costs are of two types. One type, a fixed cost $k$, is independent of the level of the damage payments. A second type is variable and proportional to the damage payments, i.e., the second type is of the form $l f \beta(P^{nd} - v(x))$ for some $l > 0$. In the following, we sometimes refer to $l$ as the "load factor." Like many of the other variables in the model, we allow both $k$ and $l$ to be random as of time 2 and represented by the random variables $\tilde{k}$ and $\tilde{l}$ with respective means $\bar{k}$ and $\bar{l}$.

To avoid making a somewhat complex model even more complex, unless otherwise indicated, all of the random variables $\tilde{q}$, $\tilde{\beta}$, $\tilde{l}$, $\tilde{f}$ are taken to be independent of each other, and also independent of all other random variables in the model.\(^9\)

We briefly digress from proceeding with the description of the model to comment on the relationship between the model setup and actual damage payments in 10b-5 and related securities litigation. First,\(^7\)

\(^7\)Shareholders who purchased shares at time 2 (and not simply shareholders who purchased after time 2 but before time 3) are also eligible to receive these damage payments because, after all, they too may have purchased the firm’s shares at the "wrong" price (where the price is assessed as "wrong" relative to the price that would have prevailed had the firm disclosed all the material information in its possession).

\(^8\)See, e.g., Ryan and Simmons (2009) for evidence on the size of the damages multiplier.

\(^9\)In more complex, and possibly more realistic, variations of the model, we might expect there to be some interdependencies among these random variables. E.g., the probability the fact finder discovers that the firm withheld material information might depend upon the rewards the fact finder obtains by this discovery. If the fact finder were an aggrieved investor who believes he purchased the firm’s shares at an artificially high price when the firm made no disclosure as of the disclosure date, the fact finder’s incentives to detect the presence of material withheld information might vary depending on the size of the damages multiplier. Alternatively, if the fact finder is the aggrieved investor’s attorney, the fact finder’s incentives to detect the presence of material withheld information could vary with the legal fees the fact finder might expect to collect.

Since there seems to be no natural limit on the extent of these, and other, potential game-theoretic issues that could create additional interdependencies among the model’s various random variables, it seems appropriate as a base case to proceed with the assumption of the independence of these random variables, and then refine that assumption in future work.
consistent with the "fraud on the market" theory of damages, investors purchasing shares between the disclosure date and the discovery date need not demonstrate reliance on the firm’s disclosures (or, if the model were embellished to include them, the firm’s financial reports) to be awarded damage payments. All that is necessary for an investor to be entitled to receive damage payments is that the investor demonstrate he purchased shares during the period in which the firm withheld MI from the capital market.

Second, that the magnitude of the assessed damages is proportional to the difference between the "artificially high" price of the firm arising from the firm having withheld MI and the "as if" price of the firm were the firm to have disclosed the MI, seems consistent with practice.

Also consistent with prevailing practice, the model defines damages in terms of the losses of investors who purchased shares between the disclosure date and the discovery date. The profits that shareholders who sold shares while the firm’s value was overstated are disregarded, even though arguments have been made that, on efficiency grounds, such shareholders should be required to disgouge their profits.

Third, the representation of materiality is "output-based" in the sense that whether the withheld information is deemed material is determined by the amount by which the price of the firm would have changed had information withheld by the firm been disclosed. This representation seems to fit well with the U.S. Supreme Court’s notion of materiality as reported, for example, in the case of Basic Inc. v. Levinson, where the Court found that "materiality depends on the significance the reasonable investor would place on the withheld or misrepresented information." That is, omitted information is considered to be material if its disclosure would have had a sufficiently significant influence on the firm’s price.

Fourth, when a firm is found to have withheld MI, the damages paid to the shareholders who purchased shares at the "artificially high" price \( P^{nd} \) established by the capital markets are compensated after the fact finder has discovered the material withholding. That is, in effect the shareholders who pay for these damages are not the shareholders of record at the time the tort (of witholding information) was committed, but rather are the shareholders of record as of (or shortly after) the time the tort is discovered. While it may be argued that better incentives to induce the firm to disclose its information would be achieved were the former set of shareholders made liable, the impracticality of identifying those former shareholders and making them pay is generally thought to rule out that alternative as infeasible. However, in section 6 below, we investigate this alternative, and we contrast the equilibrium disclosure policies that result from its adoption to the equilibrium disclosure policies derived from the conventional assignment of liability.

Fifth, many of the parameters in the model that could affect the magnitude of the tort-committing firm’s assessed damages are represented by random variables, the realizations of which are not known at the time the firm must make its disclosure decision. This comports well with the uncertainty that firms actually face concerning both the probability their withholding will be discovered and the magnitude of damages they will be assessed in the event their withholding is discovered.

Sixth, as was mentioned in the Introduction, the model introduces a distinction between information received by someone inside a firm and information received by the firm’s senior management. We incorporate this distinction formally into the analysis by positing that when value-relevant information arrives somewhere

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12 See, e.g, Posner [2007], at p. 483.
inside a firm, the firm's senior managers become aware of its arrival with probability $r$. In prior analyses, $r = 1$ always has been assumed implicitly, i.e., senior managers were assumed to be aware of all value-relevant information that arrived at their firm. We refer to the probability $r$ as the firm's internal information diffusion (iid) probability. We view the iid probability as a parameter of the model, affected by a firm's organizational design, and we generate comparative statics involving that parameter.

There would be no point in introducing the iid probability into the model were the firm to be held liable only for failing to disclose MI known by its senior management: were that the case, all that would matter would be the combined probability $p \times r$ that senior managers knew that information arrived at their firm. But, we proceed by presuming that a firm is liable for not disclosing MI possessed by someone inside the firm even if its senior managers are not aware of the information. There are good justifications for imposing such vicarious liability. Convention is one justification. The expression: the senior executives knew or should have known...is commonplace legal vernacular.13 In addition, there are both technological and economic reasons for this dictum. The technological reason is that courts, arbitrators, etc. may be able to ascertain that MI did reside somewhere inside a firm, but they may have difficulty identifying which specific parties in the organization were privy to the information. The economic reason is incentives-based: even if the courts, arbitrators, etc., could distinguish between those parties with access to the information from those parties who did not have access to the information, they could well consider it the responsibility of the firm's senior management to make sure that the senior management is aware of any MI possessed by anyone inside their organization. To do otherwise is to invite firms' senior management to seek "cover" by claiming that various information walls exist inside their organizations that prevent them from being aware of information that they would otherwise be required to disclose or, in non 10b-5 contexts, that they would otherwise be responsible for. Such defenses, if accepted, could make a firm's senior management inappropriately invulnerable to charges of wrongdoing in a broad array of both disclosure and non-disclosure related activities.

Returning now to the description of the model, we note that a firm discloses information only if all three of the following conditions holds: 1. information arrives at the firm; 2. senior managers of the firm are aware of its arrival; and 3. senior managers elect to disclose the information. In the present analysis, we do not consider the possibility that information inside the firm will be disclosed by someone other than the firm's senior management, in view of the general strictures that firms impose on their employees against making unauthorized disclosures.14 We also note that, in the model, a firm is liable for failure to disclose

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13 Appeal to the "knew or should have known" standard arises frequently in the law. For three of a plethora of examples, see:


- "Employer Stock Litigation: The Tension Between ERISA Fiduciary Obligations and Employee Stock Ownership," New York City Bar (USNW300097069v16).

- Q&A discussion by FINRA at http://www.finra.org/Industry/Compliance/RegulatoryFilings/CustomerComplaints concerning "Ten Disclosure Events" pursuant to NASD Rule 3070(a) and NYSE Rule 351(a).

14 Many firms include in their corporate code of ethics clear restrictions on employees' releasing business and other confidential information. Diamond Foods' code of ethics is representative. In it, at point 10, it is stated that:

As a condition of employment with Diamond, each employee is required to sign a confidentiality agreement. This agreement imposes an obligation upon each and every employee to protect Diamond's proprietary information, which includes such things
information only if all four of the following conditions hold: 1. someone inside the firm possessed "adverse" information circa the disclosure date ("adverse" meaning information which, if disclosed, would result in the firm’s per share price dropping below the no disclosure price $P^{nd}$); 2. the information was material; 3. the firm’s senior management failed to disclose the information for any reason (either because they deliberately withheld the information or because they were ignorant of the information); and 4. the fact finder determines that the firm withheld MI. The firm is not liable for failing to disclose "favorable" information ("favorable" meaning information which, if disclosed, would result in the firm’s per share price increasing above the no disclosure price $P^{nd}$), regardless of how big an increase in the firm’s market price would have resulted were the favorable information disclosed.\footnote{This asymmetric treatment in assessing damages associated with firms who are determined to have withheld favorable news relative to unfavorable news in practice arises because of the difficulty in determining the pertinent damaged investors, and the amount of damages they suffered, in the case of favorable news: if the failure to disclose favorable information deterred some investors from purchasing shares in the firm, how would one determine after-the-fact who those investors were, and how many shares they would have bought had the favorable news been disclosed? In contrast, when the firm withheld unfavorable news, there is no difficulty whatsoever in determining which investors were harmed after-the-fact, and how much they were harmed: the harmed investors are the investors who purchased shares at the artificially high price due to the firm’s having withheld the unfavorable news, and the amount of harm they were subject to is determined by difference between the price they paid for the shares they purchased and the price they should have paid for those shares had the unfavorable news been disclosed.}

To complete the description of the base model, we let a firm’s disclosure policy be described by a set $ND$ of the realizations of the firm’s private information $\tilde{x}$ that its senior management elects not to disclose,\footnote{Implicit in this specification is the assumption that the firm’s disclosure policy is deterministic, i.e., that the firm does not want to randomize in deciding whether or not to make a disclosure.} and we suppose that a firm chooses $ND$ so as to maximize the expected market value of the firm as of the disclosure date based on the information its senior management possesses at that time.

## 3 Definition of Equilibrium in the Base Model

Since the price of the firm in the event the firm discloses $x$ by the disclosure date is given by $v(x)$, and the prior distribution of $\tilde{x}$ is taken to be exogenous in the base model, all that is needed to define the equilibrium in the base model is: (1) a specification of the price $P^{nd}$ prevailing between the disclosure date and the discovery date in the event the firm makes no disclosure; (2) the no disclosure set $ND$; and (3) the relationship between $P^{nd}$ and $ND$.

Given the firm’s postulated objective of maximizing its expected price as of the disclosure date, and the monotonicity of the firm’s market value $v(x)$ in $x$, it is clear that if the firm prefers to disclose information $x$ it receives at the disclosure date, it also prefers to disclose any information $x' > x$ it receives at the disclosure date too. This establishes that the "no disclosure" set must be a set of the form

$$ND = \{x | x < x^c\}, \quad (1)$$

for some cutoff $x^c$. (Without loss of generality, we assume that if the firm is indifferent between disclosing its information and not disclosing it, the firm discloses.) Since the firm will disclose information $x$ only if
the price \(v(x)\) it gets by disclosing \(x\) exceeds the no disclosure price \(P^{nd}\), it follows that \(x^c = v^{-1}(P^{nd})\).

We can simplify notation somewhat by suppressing the random variable \(\tilde{x}\) and introducing the transformed random variable \(\tilde{v} = v(\tilde{x})\) with cdf \(G(v) \equiv \Pr(\tilde{v} \leq v)\), density \(g(v)\), and support \([\underline{v}, \overline{v}] \equiv [v(\underline{x}), v(\overline{x})]\).\(^{17}\)

Then, the no disclosure set \(ND\) is described by the set \(ND = \{v|v < v^c\}\), with \(v^c = P^{nd}\).

Throughout, we assume that the density \(g(\cdot)\) of \(\tilde{v}\) is continuous on its support \([\underline{v}, \overline{v}]\). Consequently, \(g(\cdot)\) is bounded. We let \(b > 0\) be such that \(g(\cdot) \leq .5b\).

As noted in footnote 7 above, if the firm made no disclosure by the disclosure date (time 2), investors who purchased shares at time 2 are among the shareholders (along with all shareholders who purchased after time 2 but before time 3) who are entitled to receive damage payments in the event the fact finder discovers that the firm withheld MI as of time 2. Investors purchasing shares at time 2 will take into consideration the expected value of those damage payments when deciding how much to pay for the firm’s shares. In the event the fact finder determines that MI \(x\) was withheld, and the damage multiplier turns out to be \(\beta\), buying a share at price \(P^{nd}\) at time 2 entitles a shareholder to the sum of:

- (i) the damage payment per share of \(\beta(P^{nd} - v(x))\) and
- (ii) a piece of the firm which - after the payment of damages to all the shareholders who bought shares between times 2 and 3 and after paying the costs of legal representation - is worth (per share) \(v(x) - f\beta(P^{nd} - v(x)) - k - l f \beta(P^{nd} - v(x))\).

So, under these circumstances (the fact finder discovered that MI was withheld) the total value of buying one share of the firm (gross of the cost of purchasing the share) is:\(^{18}\)

\[
\beta(P^{nd} - v(x)) + v(x) - \beta f(1 + l)(P^{nd} - v(x)) - k = v(x) + \beta(1 - f(1 + l))(P^{nd} - v(x)) - k. \tag{2}
\]

Several points are worth making here.

First, as was previously noted, (2) conforms to the prevailing practice that the shareholders of record as of (really, slightly after) time 3 effectively bear the cost of making the damage payments when the fact finder discovers that the firm withheld MI at time 2, since the firm they have acquired a stake in makes the damage payments at that time. This reduces the value of the firm as of time 3 in these circumstances, as the second bullet point above (2) acknowledges. And the anticipation at time 2 of the possibility of the firm having to make these damage payments at time 3 reduces how much shareholders are willing to pay for the firm at time 2. But notice that, even if - contrary to prevailing practice - the shareholders of record as of

\(^{17}\)Recalling that \(L(x)\) denotes the cdf of \(x\), it follows that \(G(v) = L(v^{-1}(v))\).

The results that follow also hold for a random variable \(\tilde{v}\) with infinite support provided that its mean \(E[\tilde{v}]\) is finite and its density \(g(\cdot)\) is continuous and bounded on its support.

\(^{18}\)Perhaps the following numerical example helps to clarify (2). Suppose \(k = l = 0\) and there are in total 100 shares of the firm. Suppose also that the price per share when \(\tilde{x} = x\) would have been \(v(x) = $30\) were \(x\) disclosed. Also suppose that if the firm does not disclose \(x\), then the no disclosure price is \(P^{nd} = 50\), that \(\beta = .6\), and that 25 shares are purchased between times 2 and 3. Then, if the fact finder discovers the firm withheld \(x\), then the damages paid per share are \(.6 \times (50 - 30) = $12\). Accordingly, the total value of the firm at time 3, after payment of damages, is \(100 \times 30 - 300 = $2700\), or \(27$/share. Alternatively, one can calculate the ex post value of the firm per share directly as in the text as: \((v(x) - \beta f(P^{nd} - v(x)) = 30 - .6 \times 25 \times (50 - 30) = $27\). If an investor who intended to purchase a share of the firm at time 2 anticipated that the firm was withholding information which, if disclosed, would result in its value per share being \(30\), and so could collect damages of \$12 per share, the investor would be willing to pay \$39=27+12 per share (or, alternatively computed via (2) as \(30+.6 \times .75 \times (50 - 30) = $39\).
time 2 instead were personally liable for making these payments (or, what amounts to the same thing, the firm at time 2), (2) still correctly describes the value of what investors as of time 2 are trying to price when contemplating the purchase of the firm’s shares then. While, in this latter case, the value of the firm at time 3 will be unaffected by any damage payments that may arise (that is, the value of the firm at time 3 will be \( v(x) \) when \( \bar{x} = x \) in this case, regardless of the fact finder’s findings), investors at time 2 will reduce the amount they are willing to pay for the firm then by the anticipated cost of the damage payments they themselves will bear were the fact finder subsequently to discover that the firm withheld MI at time 2. In short, since the size of the damage payments is unaffected by whether the shareholders of record at time 2 or at time 3 bear the damage payments, the net value to investors at time 2 of buying a share in the firm is the same whether they or the firm/shareholders of record at time 3 are responsible for the damage payments.

Notwithstanding this conclusion, we show in Section 6 below that a firm’s preferred disclosure policy may change radically when the shareholders of record at time 2, as opposed to at time 3, bear the costs of making the damage payments.

Second, (2) provides insight into when the damage payments constitute a "net wash," as detailed in the accompanying footnote.\(^{19}\)

Third, it is clear from (2) that the variable costs of legal representation are mathematically equivalent to an increase in \( f \) (from "\( f^n \)" to "\( f \times (1 + l)^n \)"), so for the analysis that follows, we will simplify notation slightly - with no loss of generality - by setting \( l = 0 \) and suitably reinterpreting \( f \) as the sum of the fraction of shares traded and the "load" the firm incurs for legal representation. For future reference, we remind the reader of this fact in the following offset:

Unless otherwise noted, henceforth "\( f^n \)" in the following represents the sum of:

1. the fraction of shares purchased between the disclosure date and the discovery date;  
2. the load factor \( l \) determining the extra payments to cover legal fees.

\(^{19}\)If \( f = 1, l = 0, \) and \( k = 0, \) the damage payments in (2) are a "net wash," in the sense that a shareholder winds up being no better off or no worse off with the damage payments than without the damage payments: the shareholder just winds up with \( v(x) \) when \( \bar{x} = x \) in any event. This should be no surprise, because if \( f = 1, \) all of the shares of the firm are bought between times 2 and 3, and so all of the firm’s shareholders are entitled to damage payments, and so the value of the firm goes down by the exact same amount that shareholders pay themselves (in the form of damage payments). Of course, if there are any frictions in the system, i.e., \( l > 0 \) or \( k > 0, \) this will not be true; in that case, even when all shares of the firm are bought between times 2 and 3, shareholders would have been better off with no damage payments than with a positive level of damage payments.

When \( (1 + l) \times f \neq 1, \) we see from (2) that the damage payments will turn out to be a "net wash," (where, here, "net" is net of the costs of legal representation) only in the knife edge case where \( v(x) = P^{\text{nd}} - \frac{k}{\beta(1-f(1+l))}, \) because in only that case will

\[
(1-f(1+l))(P^{\text{nd}} - v(x)) - k = 0.
\]

Also, notwithstanding the above discussion of "net washes," an investor is never "net" *net" better off (*"net" *net" meaning net of the costs of legal representation and also net of the cost of purchasing a share) buying a share at price \( P^{\text{nd}} \) per share when it turns out subsequently that the fact finder determines ex post that the firm withheld MI on the disclosure date, as long as the damages multiplier \( \beta \) satisfies \( \beta \leq 1. \) To see this, simply note that, netting out the price \( P^{\text{nd}} \) an investor pays per share, we see that the shareholder winds up with (*"net" *net")

\[
v(x) - P^{\text{nd}} + \beta(1-f \times (1+l)) \times (P^{\text{nd}} - v(x)) - k = (v(x) - P^{\text{nd}}) \times (1 - \beta(1-f \times (1+l))) - k. \quad (3)
\]

If \( \beta(1-f \times (1+l)) \leq 1, \) then since the firm only withholds the information \( x \) it receives when \( v(x) < P^{\text{nd}}, \) a shareholder is worse off under *these circumstances* (i.e., when the fact finder determines ex post that the firm withheld MI as of the disclosure date) even after being compensated with damage payments than he would have been had he avoided purchasing any shares of the firm: that is, \( (3) \) is negative. Of course, these circumstances are not the only ones that can occur when the firm makes no disclosure - for example, the firm might not have disclosed information because it did not receive information. In this instance, when the shareholder pays \( P^{\text{nd}} \) per share he is receiving a bargain price relative to the firm’s expected value in that circumstance (as the firm’s expected value is \( E[v(\bar{x})] > P^{\text{nd}} \) then).
Given the notational convention in (4), notice that \( f > 1 \) is possible (as is \( \bar{f} > 1 \)).

To uncover the equilibrium relationship between \( ND \) and \( P^{nd} \), we begin by assuming that investors believe the firm has adopted a no disclosure set \( ND \) defined by some cutoff \( v^c \) and then we answer the question: what do investors perceive the expected value of the firm to be, given that the firm has made no disclosure? To answer that question, we break the question up into subparts by addressing the following sequence of (sub)questions (and then reassemble the answers to these subquestions in (8) below – which is where a first-time reader may wish to jump to now):

- Given that the firm makes no disclosure, what is the probability - as perceived by investors - that the fact finder subsequently will discover that the firm withheld material adverse information as of the disclosure date? (Material adverse information is any information \( \hat{\bar{v}} < v^c - \Delta_m \).) This probability is:
  \[
  \frac{pG(v^c - \Delta_m)\bar{q}}{1 - p + p(1 - r + rG(v^c))}.
  \]

- What is the expected value of the firm (per share), given that a fact finder determines that the firm withheld material adverse information? In view of (2) and (4) above, this is given by:
  \[
  E[\hat{\bar{v}} + \hat{\beta} \times (1 - \hat{\bar{f}}) \times (P^{ND} - \hat{\bar{v}}) - \hat{\bar{k}}|\hat{\bar{v}} < v^c - \Delta_m].
  \]

- Given that the firm makes no disclosure, what is the probability - as perceived by investors - that the fact finder subsequently will discover that the firm withheld immaterial adverse information as of the disclosure date? This probability is:
  \[
  \frac{p(G(v^c) - G(v^c - \Delta_m))\bar{q}}{1 - p + p(1 - r + rG(v^c))}.
  \]

- What is the expected value of the firm, given that a fact finder determines that the firm withheld immaterial adverse information? This is:
  \[
  E[\hat{\bar{v}}|v^c - \Delta_m \leq \hat{\bar{v}} < v^c].
  \]

- Given that the firm makes no disclosure, what is the probability - as perceived by investors - that the fact finder subsequently will discover that the firm withheld favorable information as of the disclosure date? (Favorable information is any information \( \hat{\bar{v}} \geq v^c \).) This probability is:
  \[
  \frac{p(1 - G(v^c))(1 - r)\bar{q}}{1 - p + p(1 - r + rG(v^c))}.
  \]

- What is the expected value of the firm, given that a fact finder determines that the firm has withheld favorable information? This is:
  \[
  E[\hat{\bar{v}}|v^c \leq \hat{\bar{v}}].
  \]

- Given that the firm makes no disclosure, what is the probability - as perceived by investors - that the fact finder subsequently will not discover that the firm withheld information of any kind as of the disclosure date? Since the sum of the probabilities that the fact finder will find one of: material adverse
information withheld, immaterial adverse information withheld, or favorable information withheld is the sum of (5), (6), and (7), i.e., \( \frac{pq(v^c)(G(v^c)r + 1 - r)}{1 - p + p(1 - r + rG(v^c))} \), the probability that the fact finder will not discover any withholding of information is:

\[
1 - \frac{pq(G(v^c)r + 1 - r)}{1 - p + p(1 - r + rG(v^c))}.
\]

- What is the probability - as perceived by investors - that the firm’s senior managers received information as of the disclosure date, given that the firm made no disclosure and the fact finder did not discover that any information was withheld? This probability is:

\[
\frac{rG(v^c)p(1 - \tilde{q})}{1 - p + p(rG(v^c) + 1 - r)(1 - \tilde{q})}.
\]

- What is the expected value of the firm, given that the firm’s senior management made no disclosure and withheld unfavorable information but the fact finder did not discover that the firm withheld any information? This is:

\[
E[\hat{\nu}|\hat{\nu} < v^c]
\]

- What is the probability - as perceived by investors - that the firm’s senior managers did not receive information as of the disclosure date, given that the firm made no disclosure and the fact finder did not discover that any information was withheld? This probability is:

\[
1 - \frac{rG(v^c)p(1 - \tilde{q})}{1 - p + p(rG(v^c) + 1 - r)(1 - \tilde{q})}.
\]

What is the expected value of the firm, given that the firm’s senior management did not receive any information and the senior management made no disclosure and the fact finder did not discover that any information was withheld? This is:

\[
E[\hat{\nu}]
\]

Putting all these observations together, we conclude that investors will perceive the expected value of the firm ("EVF") to equal \( EVF(v^c) \), given that the firm made no disclosure and that the firm adopted the cutoff \( v^c \), where:

\[
EVF(v^c) \equiv \frac{pG(v^c - \Delta_m)\tilde{q}}{1 - p + p(1 - r + rG(v^c))} \times E[\hat{\nu} + \tilde{j} \times (1 - \hat{f}) \times (P_{1ND} - \hat{\nu}) - \tilde{k}|\hat{\nu} < v^c - \Delta_m] + \]

\[
\frac{p(G(v^c) - G(v^c - \Delta_m))\tilde{q}}{1 - p + p(1 - r + rG(v^c))} \times E[\hat{\nu}|v^c - \Delta_m \leq \hat{\nu} < v^c] +
\]

\[
\frac{p(1 - G(v^c))(1 - r)\tilde{q}}{1 - p + p(1 - r + rG(v^c))} \times E[\hat{\nu}|\hat{\nu} < \tilde{\nu}] +
\]

\[
(1 - \frac{pq(v^c)(G(v^c)r + 1 - r)}{1 - p + p(1 - r + rG(v^c))}) \times \begin{cases} 
\frac{rG(v^c)p(1 - \tilde{q})E[\hat{\nu}|\hat{\nu} < v^c]}{1 - p + p(rG(v^c) + 1 - r)(1 - \tilde{q})} + \\
\frac{rG(v^c)p(1 - \tilde{q})E[\hat{\nu}]}{1 - p + p(rG(v^c) + 1 - r)(1 - \tilde{q})} \end{cases}.
\]

With this preparatory work complete, we can define an equilibrium formally.
**Definition 1** An equilibrium of the base model consists of a "no disclosure" price \( P_{nd} \) and a no disclosure set \( ND = \{ v | v < v^c \} \) such that
\[
P_{nd} = EVF(v^c) = v^c.
\] (9)

This equation has two conditions embedded in it. On the one hand, the price \( P_{nd} \) investors assign to each of the firm’s shares in the event the firm makes no disclosure equals the expected cash flows investors will receive upon acquiring one of those shares, given that the firm uses the cutoff \( v^c \) that investors believe the firm uses. On the other hand, the cutoff the firm will use in determining whether to disclose its information is the price \( P_{nd} \) it anticipates receiving in the event it decides not to make any disclosure.

4 Analysis of the base model

We begin with two lemmas, the first of which simplifies equation (8), the solution to which determines the equilibrium of the base model, and the second of which provides sufficient conditions for this equation to have a unique solution.

**Lemma 1** The equation \( EVF(v^c) = v^c \) is equivalent to (i.e., a rearrangement of) the equation:
\[
E[\tilde{v}] - v^c = \frac{pr}{1 - pr} \int_{\tilde{v}}^{v^c} G(v)dv - \frac{pq}{1 - pr} \times \left[ (1 - \tilde{f})\tilde{\beta}\Delta_m - \tilde{k}G(\Delta_m) + (1 - \tilde{f})\tilde{\beta} \int_{v}^{v^c - \Delta_m} G(v)dv \right].
\] (10)

For readers familiar with Jung and Kwon [1988], equation (10) may be somewhat familiar. The condition defining the equilibrium cutoff in Jung and Kwon’s [1988] paper (see their equation 7, p. 149) is (in our notation) given by:
\[
E[\tilde{v}] - v^c = \frac{p}{1 - p} \int_{\tilde{v}}^{v^c} G(v)dv.
\] (11)

Note that this is identical to equation (10), when \( r = 1, k = 0 \) and any one of: \( \tilde{\beta} = 0, \tilde{f} = 1, \tilde{q} = 0 \), or \( \Delta_m \geq \tilde{v} - v \). This similarity should be expected, as Jung and Kwon [1988] coincides with the present model when there are no penalties for failing to disclose information - i.e., when all disclosures are voluntary (and so there are no overt costs for making no disclosures) - and when the senior managers of a firm are aware of all information that arrives at the firm. That any of the parameter values \( \tilde{\beta} = 0, \tilde{f} = 1, \tilde{q} = 0 \), or \( \Delta_m \geq \tilde{v} - v \) (along with \( r = 1 \) and \( k = 0 \)) give rise to this coincidence is also to be expected, as when any of these parameter conditions is satisfied, there are effectively no damages in the present model: obviously, if the damage multiplier is zero (\( \tilde{\beta} = 0 \)), there are no damages to worry about. If all of the firm’s shares are bought between the disclosure date and the discovery date (\( \tilde{f} = 1 \)), then shareholders just pay themselves damages, and the damage payments are a net wash, consistent with the discussion in footnote 19 above. If \( \tilde{q} = 0 \), then the fact finder never detects that the firm withheld MI. And if the MT is sufficiently high (\( \Delta_m \geq \tilde{v} - v \)), no instance of withholding information is ever considered material, and so no liability for withholding information arises.

Also, while we shall defer the discussion of most of the implications of (10) until after the statement of Theorem 1 below, we note here that the two probabilities \( p \) (the probability that information arrives at the firm) and \( r \) (given that information arrives at the firm, the probability that senior managers of the firm are
aware that information has arrived) do not appear in completely symmetric roles in this equation. The explanation for this is straightforward: were the firm liable for withholding MI only if its senior managers were aware of the MI, then those two probabilities would appear symmetrically in the equation, but given our assumption that the firm is liable for withholding MI whether or not the senior managers are aware of the information, symmetry of the equation in these two probabilities is not to be expected.

There are a variety of conditions that guarantee that a unique equilibrium, i.e., a unique solution to (10), exists. Two alternative sets of sufficient conditions are provided in the following lemma.

**Lemma 2** A unique equilibrium $v^c$ of the base model - i.e., a unique solution to (10) - exists, with $v^c \in (\bar{v}, \bar{v})$, under either of conditions (a) or (b) below:

(a) $\bar{f} \geq 1$

(b) $\bar{f} < 1$, $r > q(1 - \bar{f})\beta$, and $\Delta_m < \frac{1-pr}{prb}$.

Conditions (a) and (b) are straightforward: condition (a) holds when turnover in the firm’s shares between times 2 and 3 is expected to be high and the variable cost component of litigation is also high. Condition (b) holds when turnover in the firm’s shares is expected to be less than 100% and the variable cost component of litigation is low, senior managers are very likely to be aware of any information that arrives at the firm, and the materiality threshold is not too high. While there are a variety of other sufficient conditions that give rise to the existence of a unique equilibrium, the primary advantage of singling out the conditions listed in Lemma 2 is that, not only do they ensure the existence of a unique equilibrium, they prove to be sufficient to generate a wide array of comparative statics without adding any ancillary conditions (as will be seen below in Theorems 1-3).

Anticipating the next section, we now introduce a family of cdfs indexed by an "investment" variable $I \geq 0$ (described further in the next section). For a fixed $I \geq 0$, we write the cdf of a member of this family by $G(\cdot|I)$. We index the density associated with $G(\cdot|I)$ similarly, and write $g(\cdot|I)$. Consistent with our earlier notation, we let $\underline{v}$ denote the smallest of the left-endpoints of the supports of all the members of the family of cdfs $\{G(\cdot|I)|I \geq 0\}$.

We have the following definition. (In the definition, $G_I(v|I)$ refers to the partial derivative of $G(v|I)$ with respect to $I$.)

**Definition 2** The family of cdfs $\{G(\cdot|I)|I \geq 0\}$ is said to be continuously strictly ordered by stochastic dominance provided, for each $I > 0$ and each $v > \underline{v}$:

(i) $G_I(v|I) \leq 0$

(ii) $\int_\underline{v}^v G_I(v'|I)dv' < 0$

(iii) $G_I(v|I)$ and $g_I(v|I)$ both exist and are (jointly) continuous.

The crucial requirement in this definition is part (i), as it ensures that increasing the investment parameter $I$ strictly "shifts right" $G(v|I)$ in the sense of first-order stochastic dominance (i.e., if $I_1 > I_2$, then $G(v|I_1) \leq G(v|I_2)$ for all $v$). Condition (ii) ensures that this "right-shifting" is strict in the sense that it occurs to some extent over any nondegenerate interval $(\underline{v}, v)$; and condition (iii) ensures that this right-shifting of the distributions occurs "continuously."

One example (among many) of a family of cdfs that is continuously strictly ordered by stochastic dominance is the family of cumulative distribution functions corresponding to the set of uniform random variables
\{\hat{v}_I | I > 0\}$ where $\hat{v}_I$ is uniformly distributed on $[0, 1]$.

The following predictions emerge from the base model.\(^{21}\)

**Theorem 1** (The Comparative Statics of the Equilibrium Cutoff) Assume that one of conditions (a) or (b) of Lemma 2 holds, as appropriate. Then, the equilibrium no disclosure price/equilibrium cutoff is:

(a) strictly decreasing in the MT $\Delta_m$ when $(1 - \hat{f})\beta\Delta_m - \hat{k} > 0$ and strictly increasing in the MT $\Delta_m$ when $(1 - \hat{f})\beta\Delta_m - \hat{k} < 0$;

(b) strictly increasing in the damages multiplier $\beta$ when $\hat{f} < 1$ and strictly decreasing in the damages multiplier when $\hat{f} > 1$;

(c) strictly increasing in the probability $\bar{q}$ the fact finder discovers that the firm withheld MI when if $(1 - \hat{f})\beta\Delta_m > \hat{k}$ and strictly decreasing in this probability when $\hat{f} > 1$;\(^{22}\)

(d) strictly decreasing in the expected fraction $\hat{f}$ of shares purchased between times 2 and 3;

(e) strictly decreasing in the expected load factor $\bar{l}$;

(f) strictly decreasing in the expected "fixed" legal costs $\bar{k}$;

(g) strictly decreasing in the probability $p$ that information arrives at the firm;

(h) strictly decreasing in the internal information diffusion probability $r$;

(i) strictly increasing in the "investment" variable $I$ when the family $\{G(\cdot | I) | I \geq 0\}$ is continuously strictly ordered by stochastic dominance provided additionally: $\frac{(\bar{v} - v)(1 - pr)}{4pr} > \Delta_m$ and there exists a positive constant $B$ such that $|G_I(\cdot | I)| < B.$\(^{23}\)

To explain Theorem 1, recall that the equilibrium cutoff is the same as the firm’s price in the event that it makes no disclosure, so a change in any parameter that results in the firm’s equilibrium no disclosure price increasing (resp., decreasing) is tantamount to the firm reducing (resp., expanding) the circumstances under which the firm discloses its private information. It follows that Theorem 1(a) can be interpreted as showing that a firm’s propensity to disclose information is nonlinear, and in fact, non-monotonic in the MT $\Delta_m$. When a firm’s expected fixed legal costs associated with defending itself in 10b-5 litigation are positive ($\hat{k} > 0$), the equilibrium cutoff is always increasing in $\Delta_m$ at $\Delta_m = 0$ (since $(1 - \hat{f})\beta\Delta_m - \hat{k} < 0$ when

\(^{20}\)To see this, just note that $G(v|I) = \frac{v}{\bar{f}}$ for $v \in [0, I]$ and $G(v|I) = 1$ for $v > I$, so for all $v' > 0$ and all $I > 0$, $f' G_I(v|I) dv = -\int_0^{\min(v', I)} \frac{v'}{\bar{f}} dv = -\frac{\min(v', I)^2}{2\bar{f}} < 0$.

\(^{21}\)In the statement of Theorem 1, as well as the other theorems in the paper, the qualification "as appropriate" in reference to conditions (a) or (b) of Lemma 2, is intended to refer to situations like the following. Theorem 1(b) below shows that how the equilibrium cutoff changes with increases in the damages multiplier depends on whether $\hat{f} < 1$ or $\hat{f} > 1$. Condition (a) of Lemma 2 requires $\hat{f} \geq 1$ to hold, whereas condition (b) of Lemma 2 requires, among other things, $\hat{f} < 1$. Thus, the "as appropriate" qualification here means that the proof that the equilibrium cutoff is increasing in the damages multiplier when $\hat{f} < 1$ is established when condition (a) of Lemma 2 holds, whereas the proof that the equilibrium cutoff is decreasing in the damages multiplier is established when condition (b) of Lemma 2 holds.

As another example of the intended meaning of "as appropriate," consider Theorem 1(a), which establishes that whether the equilibrium cutoff is increasing or decreasing in the materiality threshold depends on whether $(1 - \hat{f})\beta\Delta_m - \hat{k}$ is positive or negative. $(1 - \hat{f})\beta\Delta_m - \hat{k}$ is positive only when $\hat{f}$ is less than 1, but $(1 - \hat{f})\beta\Delta_m - \hat{k}$ can be negative both when $\hat{f} > 1$ and when $\hat{f} < 1$. So, "as appropriate" here means that when $(1 - \hat{f})\beta\Delta_m - \hat{k}$ is positive, condition (b) of Lemma 2 is assumed to hold, whereas when $(1 - \hat{f})\beta\Delta_m - \hat{k}$ is negative, either condition is allowed to hold. For other parts of Theorem 1, such as any of Theorem 1(d) - 1(h), it does not matter which of condition (a) or condition (b) of Lemma 2 holds, as long as one of those conditions holds.

\(^{22}\)Note that there is a "hole" in this characterization, as there is a region in which $(1 - \hat{f})\beta\Delta_m < \hat{k}$ and $\hat{f} < 1$ for which this part of the theorem does not report comparative statics. As inspection of the proof of this result shows, in the omitted region of parameter space, comparative statics appear to be ambiguous.

\(^{23}\)The requirement that there exists a positive constant $B$ such that $|G_I(\cdot | I)| < B$ can be dispensed with if we replace the assumption that $\frac{(\bar{v} - v)(1 - pr)}{4pr} > \Delta_m$ with the assumption that $\Delta_m$ is sufficiently small (but positive).
\( \Delta_m = 0 \). That is, a firm is less likely to disclose some of the private value-relevant information it receives if what is considered material increases from zero (all information is considered material) to some positive amount (withholding some information is not considered material). This is to be expected: when firms obtain an increased opportunity to withhold some information from capital markets without facing legal reprisals for doing so, they exploit that opportunity by disclosing their private information less often. But, as Theorem 1(a) also reports, there is an upper bound to this effect: if materiality thresholds become too high (when \( \Delta_m \geq \frac{k}{(1-f)\beta} \), if \( \bar{f} < 1 \)), further increases in the MT cause expected value-maximizing firms to begin disclosing more information (i.e., the equilibrium cutoff decreases in \( \Delta_m \)). Why? Because MTs are public information, and investors alter their interpretation of the firm’s nondisclosure as the MT changes. When already high materiality thresholds are increased further, investors respond to firms which, they know, can fail to disclose a substantial amount of information with impunity (because the MT is very high) by assigning them a lower value when they don’t disclose information. If, contrary to the model, firms somehow had the opportunity to privately (i.e., without investors’ knowledge) increase the materiality thresholds they were subject to, then firms would invariably exploit this opportunity by withholding from investors even more unfavorable information that they receive. But, when materiality thresholds are public, increasing already high materiality thresholds causes investors to reinterpret the firms’ failure to disclose information - specifically, reinterpret their nondisclosures more unfavorably - and rational firms respond to investors’ altered interpretations by disclosing information more often.

Theorem 1(b) reports that changes in the level of the damages multiplier also has nonlinear, and non-monotonic, effects on the incentives of a firm to disclose its private information. If \( \bar{f} < 1 \), increases in the damages multiplier causes firms to withhold information more often. On its face, this is a surprising result: increases in the damages multiplier increase the penalty/liability the firm must pay for withholding information, and it would seem that firms would respond by withholding information less often. In a variation of the model (not presented here) where it is supposed that only investors who purchase a firm’s shares after the disclosure date (time 2) are entitled to damages, but investors who buy shares at time 2 are not eligible to receive the damage payments, then the typical, expected result obtains: increases in the damages multiplier uniformly increases the incentives of a firm to disclose its information at time 2. But, in the present model - and in practice - the shareholders of record at time 2 are also entitled to receive damage payments. The prospect of receiving damage payments increases the amount investors are willing to pay for the firm’s shares (holding the no disclosure price fixed) and, as is evident from (2), increasing the damages multiplier increases the amount the investors are willing to pay for those shares, as long as the firm’s expected variable payout is less than one ( \( \bar{f} < 1 \)). When \( \bar{f} > 1 \), this effect reverses itself: then, the payouts to other shareholders and the payouts in variable legal fees (recall the convention (4)) become so large that increases in the damages multiplier decrease the value investors at time 2 are willing to pay for the firm’s shares in the event the firm makes no disclosure.

The observation in Theorem 1(b) that increases in the damages multiplier need not increase firms’ propensity to disclose information is consistent with Rogers and van Buskirk’s [2009] finding that firms that undergo a “litigation event” need not be more disposed to disclose information after the event. Such firms would seem to be more sensitive than firms in general to the potential liability they may generate by failing
to disclose MI - that is, they may perceive the damages multiplier to be higher than firms that have not been subject to a litigation event. But, as Theorem 1(b) reports, firms need not respond to a perceived increase in the damages multiplier by disclosing information more often.

As Theorem 1(c) reports, an increase in the probability that the fact finder detects that the firm withheld MI in general has an ambiguous effect on the firm’s decision to disclose the information it receives. This is also, at first blush, surprising. Again, in an unreported version of the model where the shareholders of record in the firm after time 2, but not at time 2, are eligible to receive damage payments, the anticipated result happens: increases in the probability of detecting the withholding of information leads to a decrease in the firm’s incentives to withhold MI. The ambiguous effect arises because, as was true in Theorem 1(b), investors at time 2 are ambivalent about having the fact finder be successful in detecting that the firm withheld MI, because they are themselves recipients of the damage payments in the event the fact finder discovers material withholding. When the payouts to others become too large ($f > 1$), an individual investor contemplating the purchase of the firm’s shares at time 2 becomes less willing to pay for the firm’s shares as the fact finder’s effectiveness increases. But, if the firm does not have to pay fixed costs to settle claims ($k = 0$), and the variable payout to others is not too high ($f < 1$), the amount investors are willing to pay for a share of the firm when the firm makes no disclosure is increasing in probability the fact finder detects the firm withheld MI.

The explanation for Theorem 1(d) is clear: as more investors are anticipated to buy the firm’s shares between the disclosure date and the discovery date and hence become eligible to collect damages from a firm that withholds MI, a firm becomes less inclined to withhold the information it receives. Virtually the same comments apply with respect to increases in the load factor, given the convention (4), which explains Theorem 1(e). The expected fixed costs of litigation $\tilde{k}$ are a deadweight loss on the firm, which the firm can reduce by disclosing its information more often. This explains Theorem 1(f). Theorem 1(g) is true for the same reason that it is true in the original Jung and Kwon[1988] model: if investors know that information arrives at a firm more often, they will become more skeptical of a firm that does not disclose information, and they reduce the firm’s price accordingly. Theorem 1(h) is similar: even though failure to disclose MI that arrives at a firm generates liability for a firm when senior managers are not aware of its arrival, investors recognize that if the senior managers of a firm are frequently ignorant of information that arrives at their shop (i.e., the iid probability $r$ is small), investors will not discount the price of the firm’s shares much when the firm makes no disclosure, because they recognize that the firm’s failure to disclose information is not evidence of a "lemon’s problem," i.e., is not evidence that the firm’s senior managers are deliberately choosing to withhold bad information from the capital markets. As the iid probability $r$ rises, the reverse is true. Finally, Theorem 1(i) - that as a firm increases the level of its investment - which "right-shifts" the distribution of $\tilde{v}$ (which is what investors are pricing) - then investors know that the firm’s nondisclosure is not such bad news, because high values of $\tilde{v}$ are more likely to occur whether or not the firm makes a disclosure. A value-maximizing firm can exploit investors’ increasingly sanguine response to the firm’s nondisclosure by withholding information in its possession more often.

Theorem 1’s results have implications for the size of the firm’s price reaction to the nondisclosure of information at time 2. Under the hypothesis of Lemma 2(b), there will be a negative price reaction
attending the lack of a disclosure by the firm at time 2, assuming the parameters determining the litigation costs ($\bar{k}$ and $\bar{l}$) are sufficiently small. To see this, first note that when $\bar{k} = \bar{l} = 0$, then even though there are damage payments in the model, those damage payments are between groups of shareholders in the firm, and hence the damage payments are zero sum. Therefore, the price of the firm at time zero is just the firm’s unconditional expected value $E[\bar{v}]$. When $\bar{k}$ and $\bar{l}$ are sufficiently small, it follows that the price of the firm at time zero will be nearly $E[\bar{v}]$ too. Then, notice that, under the conditions of Lemma 2(b), $E[\bar{v}] > v^c$, since $\bar{k} \geq 0$ and $r > \bar{q}(1 - \bar{f})\bar{\beta}$ together imply:

$$RH\bar{S}(10)$$

$$\geq \frac{pr}{1 - pr} \int_{v^c}^{v^e} G(v) dv - \frac{p\bar{q}(1 - \bar{f})\bar{\beta}}{1 - pr} \times \left[ \Delta_m G(v^c - \Delta_m) + \int_{v^e - \Delta_m}^{v^e} G(v) dv \right]$$

$$\geq \frac{pr}{1 - pr} \left( \int_{v^c}^{v^e} G(v) dv - \Delta_m G(v^c - \Delta_m) - \int_{v^e - \Delta_m}^{v^e} G(v) dv \right)$$

$$= \frac{pr}{1 - pr} \left( \int_{v^e - \Delta_m}^{v^e} G(v) dv - \Delta_m G(v^c - \Delta_m) \right)$$

$$= \frac{pr\Delta_m}{1 - pr} (G(\psi) - G(v^c - \Delta_m))$$

$$> 0,$$

where in the last equality $\psi$ is a number (by the mean value theorem for integrals) in the interval $(v^c - \Delta_m, v^e)$.

From this observation, the comparative statics in Theorem 1 translate into statements about the size of the negative price reaction to the firm’s nondisclosure at time 2. For example, Theorem 1(a) implies that the negative price reaction will be bigger as the MT $\Delta_m$ increases if $(1 - \bar{f})\bar{\beta}\Delta_m - \bar{k} > 0$ and the negative price reaction will be smaller as the MT $\Delta_m$ increases when $(1 - \bar{f})\bar{\beta}\Delta_m - \bar{k} < 0$. The other comparative statics reported in Theorem 1 similarly can be translated into statements about the size of the price reactions to the firm’s nondisclosure at time 2.

While the Appendix contains the formal proofs of Theorem 1, we can acquire an intuitive understanding of how many of Theorem 1’s results are derived by considering the special case where all withheld information is considered material, i.e., $\Delta_m = 0$, and the fixed legal costs $\bar{k}$ are negligible: $\bar{k} = 0$. Then, it is evident that equation (10) reduces to:

$$E[\bar{v}] - v^c = \frac{pr\Delta_m}{1 - pr} \times \int_{\underline{\psi}}^{v^e} G(v) dv.$$  \hspace{1cm} (12)

This last equation is even more similar in form to the equation (11) defining the cutoff in the original Jung and Kwon [1988] model. Both these equations are of the form:

$$E[\bar{v}] - v^c = \alpha \int_{\underline{\psi}}^{v^c} G(v) dv,$$  \hspace{1cm} (13)

for some parameter $\alpha$: in the case of Jung and Kwon[1988], $\alpha = \frac{p}{1 - pr}$; in the case of (12), $\alpha = \frac{p(1 - \bar{q}(1 - \bar{f})\bar{\beta})}{1 - pr}$.

As was noted in connection with the discussion of Theorem 1(g) above, the intuitive explanation for why

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24 We know that $\psi \in (v^c - \Delta_m, v^e)$, as opposed to simply $\psi \in [v^c - \Delta_m, v^e]$, since $G(v)$, as a cdf, is strictly increasing over its support.
the equilibrium cutoff/no disclosure price both here and in Jung and Kwon [1988] declines in the probability 
$p$ that the firm receives information is that investors interpret a firm’s nondisclosure more skeptically the 
higher the probability that the firm receives information, and the firm’s equilibrium response to investors' 
increased skepticism is to disclose the information it receives more often. But, the formal explanation both 
here and in Jung and Kwon [1988] (at least, when $k = \tilde{k} = m = 0$) is that the solution $v^c = v^c(\alpha)$ to (13) must 
be decreasing in $\alpha$, and when $\alpha = \frac{p}{1 - p}$, an increase in $p$ is tantamount to an increase in $\alpha$.$^{25}$ This same 
formal logic applies to equation (12), not just for $p$, but for any parameter that increases the parameter 
$\alpha = \frac{v(r - \bar{v}(1 - \tilde{\beta}))}{1 - pr}$ in (13).$^{26}$

5 Expanding the model to include endogenous investment

All of the analysis so far has been undertaken assuming that what investors cared about - $\tilde{v}$— was specified by 
an exogenously given distribution. This is clearly not the case in practice: investment choices made by the 
firm will affect this distribution.$^{27}$ This section extends the analysis undertaken previously so as to account 
for the, in practice, endogenous nature of the distribution of $\tilde{v}$. We shall show that all of the predictions of 
the model with exogenous investment translate immediately into predictions of the model with endogenous 
investment. Specifically, we shall show that a change in any exogenous parameter that results in the firm’s 
"no disclosure" price increasing (resp., decreasing) will lead to a lower (resp., higher) equilibrium level of 
investment and a lower (resp., higher) equilibrium expected value for the firm.

As we anticipated in the previous section both in the definition of a family of cdfs continuously strictly 
ordered by stochastic dominance and in Theorem 1(i), we now suppose that at time 0, the firm under 
investigation now privately chooses an investment level $I \geq 0$. This investment level affects the cdf $G(v|I)$ 
of $\tilde{v}$. We shall suppose throughout this section that the family of cdfs $\{G(\cdot|I)|I \geq 0\}$ is continuously strictly 
ordered by stochastic dominance. Among other things, this implies that the mean $E[\tilde{v}|I] = \int_{v} vG(v|I)dv$ 
increases in $I$. We take the time 0 cost of making the investment $I$ is $I$ itself. We ignore discounting.

The firm’s choice of investment $I$ is assumed to be private information to the firm. Accordingly, we 
must distinguish between the investment choice the firm actually undertakes from the investment investors 
conjecture the firm has undertaken. We (continue to) write $I$ for the firm’s actual investment choice, and we 
introduce the notation $\hat{I}$ to represent investors’ conjectures about the firm’s investment choice. (Of course, 
in equilibrium, these will be one and the same.)

Holding fixed investors’ conjecture $\hat{I}$, along with all the parameters of the model $(\Delta_m, f, \tilde{k}, \tilde{\beta}, \bar{q}, p, r)$, by

---

$^{25}$To see this, just differentiate (13) totally with respect to $\alpha$ to get

$$\frac{dv^c(\alpha)}{d\alpha} = \int G(v)G(v)dv \left(1 + aG(v^c(\alpha))\right).$$

$^{26}$When $\Delta_m > 0$, the preceding arguments do not apply, and a somewhat more involved argument must be invoked. Please 
see the Appendix for details.

$^{27}$The idea that financial reporting and disclosure decisions should be evaluated in terms of their real effects, and in particular, 
in terms of their effects on a firm’s equilibrium investment choices, was initiated by Kanodia: see, e.g., Kanodia [1980], Kanodia 
and Lee [1998], and Kanodia and Mukherji [1996].

---
appealing to (10), we can immediately conclude that the threshold $\hat{v}^c = v^c(\hat{I})$ must solve:

$$
E[I][\hat{v}] - \hat{v}^c = \frac{p\tilde{\gamma}}{1 - p}\times \left[ (\tilde{f}\tilde{\beta}\Delta_m + \tilde{\kappa}) \times G(\hat{v}^c - \Delta_m|\hat{I}) + \tilde{f}\tilde{\beta} \times \int_{v}^{\hat{v}^c - \Delta_m} G(v|\hat{I})dv \right] + \frac{p\tilde{\gamma}}{1 - p}\int_{v}^{\hat{v}^c} G(v|\hat{I})dv.
$$

(14)

When deciding what value of investment $I$ actually to select, the firm will take $\hat{v}^c$ as given. It follows that, given $\hat{I}$ and $I$, the expected value of the firm is given by:

$$
V(\hat{v}^c, I) = (1 - p + p(1 - r + rG(\hat{v}^c|I)))\hat{v}^c + pr(1 - G(\hat{v}^c|I))E[\hat{v}|\hat{v} > \hat{v}^c, I]
$$

$$
= (1 - p + p(1 - r + rG(\hat{v}^c|I)))\hat{v}^c + pr\int_{\hat{v}^c}^{\hat{v}} v g(v|I)dv.
$$

(15)

So, the firm’s expected value net of the cost of the investment $I$ is given by:

$$
W(\hat{v}^c, I) \equiv V(\hat{v}^c, I) - I.
$$

(16)

Given investors’ conjecture $\hat{I}$ and the associated implied cutoff $\hat{v}^c = v^c(\hat{I})$, the firm’s preferred investment choice is given by the first-order condition $\frac{\partial}{\partial I}W(\hat{v}^c, I) = 0$, i.e., which in view of (15), is given by:

$$
prG_1(v^c(\hat{I})|I)v^c(\hat{I}) + pr\int_{v^c(\hat{I})}^{\hat{v}} vg(v|I)dv - 1 = 0.
$$

(17)

The following definition formalizes the notion of an equilibrium in this setting.

**Definition 3** An equilibrium consists of an investment level $I$ and a no disclosure price $\hat{v}^c$, such that,

(i) given the price $\hat{v}^c$ :

$$
I \in \arg \max_{I \geq 0} W(\hat{v}^c, I);
$$

(18)

(ii) the no disclosure price $\hat{v}^c$ is given by:

$$
\hat{v}^c = v^c(I).
$$

In words, an equilibrium must satisfy two conditions: first, taking investors’ conjecture $\hat{I}$ about the firm’s investment - as well as the "no disclosure" price $\hat{v}^c = v^c(\hat{I})$ implied by that conjecture $\hat{I}$ - as given, the firm’s preferred investment level $I$ maximizes the expected value of the firm net of the cost of making the investment. Second, the firm’s "no disclosure" price $\hat{v}^c$ based on investors’ conjecture $\hat{I}$ about the firm’s investment is the "correct" no disclosure price in that it is the "no disclosure" price that corresponds to the investment level the firm actually adopts.

While we are much more interested in characterizing features on an equilibrium rather than addressing issues related to the existence of equilibrium, the characterization of an equilibrium is not of much use unless it is known that an equilibrium exists. So, we digress briefly to provide one set of sufficient conditions that ensures that an equilibrium exists. To present these sufficient conditions, we introduce one more definition.

**Definition 4** The family of cdfs $\{G(\cdot | I) | I \geq 0\}$ is said to possess the concavity of distribution function condition (CDFC) if $G(\cdot | I)$ is convex in $I$.

The concavity of distribution function condition amounts to the stochastic equivalent of decreasing returns to scale.\textsuperscript{28} It, or variations of it, has been used in various information economics studies, including, for example in the formal study of agency relationships (see, e.g., Grossman and Hart [1981]).

\textsuperscript{28}This condition ensures that for any strictly increasing function of $v$, say $i = i(v)$, $v \in [\underline{v}, \hat{v}]$, the expectation $J(I) \equiv \int_{\underline{v}}^{\hat{v}} i(v)dG(v|I)$ is strictly concave $I$. 

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One set of conditions sufficient for the existence of an equilibrium is the following.

**Lemma 3** If:

1. the family of cdfs \( \{G(\cdot | I) | I \geq 0 \} \) is continuously strictly ordered by stochastic dominance;
2. the family of cdfs \( \{G(\cdot | I) | I \geq 0 \} \) possesses the concavity of distribution function condition; and
3. at least one of the assumptions of Lemma 2 hold;

then, an equilibrium exists when investment is endogenous.

(The proof of the lemma is not relied on elsewhere in the manuscript. Accordingly, its proof is omitted.)

Theorem 2 below summarizes the comparative statics of firms’ equilibrium investment choices generated by this expanded model.

**Theorem 2** (The Comparative Statics of Equilibrium Investment) Assume that at least one of conditions (a) or (b) of Lemma 2 hold, as appropriate. When investment is endogenous, the equilibrium level of investment is:

(a) strictly increasing in the MT \( \Delta_m \) when \( (1 - \bar{f})\bar{\Delta}_m - \bar{k} > 0 \) and strictly decreasing in the MT \( \Delta_m \) when \( (1 - \bar{f})\bar{\Delta}_m - \bar{k} < 0 \);

(b) strictly decreasing in the damages multiplier \( \bar{\beta} \) when \( \bar{f} < 1 \) and strictly increasing in the damages multiplier \( \bar{\beta} \) when \( \bar{f} > 1 \);

(c) strictly decreasing in the probability \( \bar{q} \) the fact finder discovers that the firm withheld MI when \( (1 - \bar{f})\bar{\Delta}_m > \bar{k} \) and strictly increasing in this probability when \( \bar{f} > 1 \);

(d) strictly increasing in the fraction \( \bar{f} \) of shares purchased between the disclosure date and the discovery date;

(e) strictly increasing in the "load" factor \( \bar{l} \);

(f) strictly increasing in the expected "fixed" legal costs \( \bar{k} \);

(g) strictly increasing in the probability \( \bar{p} \) that information arrives at the firm;

(h) strictly increasing in the internal information diffusion probability \( \bar{r} \).

The similarities of the results reported in Theorem 2 to the results reported in Theorem 1 are to be noted. In each and every case, a change in any parameter that increases the firm’s equilibrium no disclosure price in the version of the model with exogenous investment reduces the firm’s equilibrium investment level in the version of the model with endogenous investment. Why? Because when the firm makes no disclosure, the firm does not have to "work" to be rewarded by the capital markets for the firm’s market value that results from the firm’s nondisclosure. So, the firm responds to a change in any parameter that results in the firm’s no disclosure price increasing - and hence results in the firm not being forced to "work" by the capital markets to obtain that increasing no disclosure price - by reducing how much investment it makes. Symmetric comments apply to changes in parameter values that result in the firm’s no disclosure price declining.

The next theorem establishes that there are also parallels between a firm’s equilibrium level of investment and the firm’s equilibrium expected market value net of the cost of the investment.

**Theorem 3** (The Comparative Statics of Equilibrium Expected Firm Value) Assume that at least one of conditions (a) or (b) of Lemma 2 hold, as appropriate. When investment is endogenous, equilibrium expected firm value net of the cost of investment is:
(a) strictly increasing in the MT $\Delta_m$ when $(1 - \tilde{f})\beta \Delta_m - \tilde{k} > 0$ and strictly decreasing in the MT $\Delta_m$ when $(1 - \tilde{f})\beta \Delta_m - \tilde{k} < 0$;

(b) strictly decreasing in the damages multiplier $\tilde{\beta}$ when $\tilde{f} < 1$ and strictly increasing in the damages multiplier $\tilde{\beta}$ when $\tilde{f} > 1$;

(c) strictly decreasing in the probability $\tilde{q}$ the fact finder discovers that the firm withheld MI when $(1 - \tilde{f})\beta \Delta_m > \tilde{k}$ and strictly increasing in the probability $\tilde{q}$ when $\tilde{f} > 1$;

(d) strictly increasing in the fraction $\tilde{f}$ of shares purchased between the disclosure date and the discovery date;

(e) strictly increasing in the fraction $\tilde{f}$ of shares purchased between the disclosure date and the discovery date;

(f) strictly increasing in the "load" factor $\tilde{l}$;

(g) strictly increasing in the probability $p$ that information arrives at the firm;

(h) strictly increasing in the internal information diffusion probability $r$.

The similarities of the results reported in Theorem 2 to the results reported in Theorem 3 are to be noted. In each and every case, a change in any parameter that increases the firm’s equilibrium investment level increases the expected value of the firm net of the cost of the investment. Why? Because when investors do not get to observe a firm’s actual investment choice, and instead have to make inferences about the firm’s investment choice, firms in equilibrium invest too little relative to first-best. Consequently, a change in any parameter that leads the firm to invest more invariably increases the firm’s expected market value, both because the firm’s original (pre-parameter change) investment level was too low (relative to first-best) and because, with the change in the parameter value, there is no possibility that the firm will overinvest (relative to first-best). Symmetric comments apply with respect to a change in any parameter that leads the firm to invest less: such a change invariably reduces the firm’s expected market value.

Theorem 3(a) provides some insight into the "optimal" level of the MT, at least from a single firm’s perspective, for some parameter values. When $\tilde{f} > 1$, the optimal MT is always zero because Theorem 3(a) reports that the expected value of the firm is decreasing in $\Delta_m$ whenever $(1 - \tilde{f})\beta \Delta_m - \tilde{k} < 0$, and this last inequality is satisfied by all $\Delta_m \geq 0$ when $\tilde{f} > 1$. When $\tilde{f} < 1$ and $\tilde{k} = 0$, the optimal materiality level is positive, because Theorem 3(a) reports that expected firm value is increasing when $(1 - \tilde{f})\beta \Delta_m - \tilde{k} > 0$ (but, Theorem 3(a) does not imply that the optimal MT is infinite when $\tilde{k} = 0$ and $\tilde{f} < 1$, because the theorem only applies (recall condition (b) of Lemma 2) to those materiality thresholds bounded above by $\frac{1 - p_{pr}}{p_{pr}}$). Whether the optimal MT is positive when $\tilde{k} > 0$ and $\tilde{f} < 1$ is not clear from Theorem 3(a), because expected firm value is locally decreasing in the vicinity of $\Delta_m = 0$, but expected firm value then "turns around" and increases in the MT for $\Delta_m > \frac{\tilde{k}}{(1 - \tilde{f})\beta}$. Results from examining optimal materiality thresholds when $\tilde{v}$ is uniformly distributed (not reported here) demonstrate that the optimal materiality threshold are sometimes, but not always, positive.

As the discussion in the paragraph immediately following the statement of Theorem 3 made clear, the primary explanation for what determines the optimal MT is what MT drives down the firm’s no disclosure price the most. When the no disclosure price at a particular MT is high, the maximum expected value of the firm is low and vice versa. In particular, whether the optimal MT is zero depends on whether
subjecting the firm to penalties for withholding any information it receives drives down the MT more than
does increasing the MT - which increases investors’ skepticism regarding the reason for the firm’s failure to
disclose information.

Other parts of Theorem 3 provides some insights into the optimal value of the model’s other parameters, as
well. The same economic forces are in effect for any of these parameters: the optimal value of the parameter
is one that serves to drive down the firm’s no disclosure price to the lowest possible level. Accordingly, the
intuition provided following the statement of Theorem 1 regarding how changes in a parameter change the
firm’s no disclosure price translate immediately into statements about what determines the optimal value of
that parameter.

6 Alternative Assignment of Liability for Withholding Material
Information

In the preceding sections of the paper, we have assumed, as is the case in practice, that damages are paid
by the firm after the fact finder deems the firm to have withheld MI and so, as we noticed previously, the
damages are effectively paid by the shareholders of record after the fact finder discovers this withholding.
While this is prevailing practice, it might not seem to be ideal, since the tort/withholding occurs under the
watch of shareholders of record at the disclosure date, and does not occur under the watch of shareholders
at the discovery date. To the extent that the shareholders of record as of a particular time can influence the
firm’s disclosure behavior at that time, it might seem preferred to have the cost of withholding information
(the damage payments) be born by those shareholders who benefit from the withholding (in the form of a
higher time 2 share price).

We evaluate this deviation from the prevailing practice of assigning damages for withholding information
not by examining the normative implications of this change, but rather by considering the change’s positive
implications - i.e., by making predictions as to how this change would alter the firm’s preferred disclosure
policy.

Even though the remark made above in reference to (2) that the value to an investor contemplating
the purchase of the firm’s shares at time 2 is independent of whether the firm’s shareholders of record at
time 2 or the firm’s shareholders of record at time 3 are responsible for damage payments that derive from
the firm’s having withheld MI at time 2 is valid, it does not follow that the firm’s preferred disclosure
policy is unaffected by this reassignment of damages. The reason is: a sensible examination of this policy
change requires reconsideration of the firm’s objective function in choosing its disclosure policy. Previously,
we posited that the firm’s disclosure policy be aimed at maximizing the firm’s expected time 2 market
value. When the firm’s shareholders at time 2 did not personally bear the costs of liability for withholding
information, that specification of the firm’s objective function seems unassailable. But, when the shareholders
at time 2 do bear this liability, it seems appropriate to modify the firm’s objective function so as to entail
maximizing the expected price of the firm at time 2 net of the expected costs of any liability that derives
from the firm’s failure to disclose MI in its possession at that time. As the analysis that follows will make
clear, this natural change in the firm’s objective function attending this reassignment of liability sometimes
- but not always - results in the firm's preferred disclosure policy changing radically.

Because the main goal of this section is to examine what changes, if any, occur in the firm's preferred disclosure policy as a consequence of this shift in the assignment of liability and the attendant shift in the firm's objective function, we confine attention to the case where the firm's investment is taken as exogenous. We justify this restriction by recognizing that, while, when the firm's investment is endogenous, the level of the firm's preferred investment depends on the firm's disclosure policy, the form of the firm's preferred disclosure policy does not change when the firm's investment switches from being exogenous to being endogenous.

All other aspects of the model remain as before: when the firm receives information \( \hat{v} = v \), it must decide whether to disclose or withhold that information. Since the firm can compute both the "no disclosure" price \( P^{nd} \) that will prevail if it withholds its information and the "disclosure price" \( v \) that will prevail if it discloses its information, the firm can compute - if it decides to withhold its information - whether that withholding will be considered material if the fact finder subsequently detects that the firm withheld information. That is, the firm can calculate whether \( P^{nd} - v \) is bigger or smaller than \( \Delta_m \). If, based on its calculation, the firm determines that the withholding would be considered immaterial, then its decision to withhold or disclose the information will be based on whether \( v \) is bigger or smaller than \( P^{nd} \). That is, letting \( ND^{im} \) denote the set of \( v \)'s that the firm prefers to withhold that it considers immaterial:

\[
ND^{im} = \{ v | v < P^{nd} \} \cap \{ v | P^{nd} - v < \Delta_m \}.
\]  

If the firm considers \( v \) to be material, then it recognizes that it/its time 2 shareholders will be assessed damages if the fact finder discovers that it withheld \( v \). If it withholds \( v \), then - since the ex ante probability the fact finder will detect that the firm withheld MI is \( \bar{q} \) - the expected value of each of its shares, net of the expected liability for withholding MI, is given by:

\[
\bar{q}(v - \bar{\beta}\bar{f}(P^{nd} - v) - \bar{k}) + (1 - \bar{q})P^{nd}.
\]  

If it discloses \( v \), it just gets \( v \). Its disclosure decision will be based on whether \( v \) is bigger or smaller than \( P^{nd} \). That is, letting \( ND^{m} \) denote the set of \( v \)'s that the firm prefers to withhold that it considers material:

\[
ND^{m} = \{ v | v < \bar{q}(v - \bar{\beta}\bar{f}(P^{nd} - v) - \bar{k}) + (1 - \bar{q})P^{nd} \} \cap \{ v | P^{nd} - v \geq \Delta_m \}.
\]  

To proceed, we subdivide the analysis depending on the relative sizes of the various exogenous parameters \( \bar{q}, \bar{\beta}, \bar{f}, \bar{k} \). We start by supposing that

\[
1 > \bar{q}(1 + \bar{\beta}\bar{f}) \quad (22).
\]

When (22) holds, notice that \( ND^{m} \) can be rewritten (after some rearrangement) as:

\[
ND^{m} = \{ v | v < P^{nd} \} \cap \{ v | v \leq P^{nd} - \Delta_m \}.
\]  

If (22) and

\[
\frac{\bar{q}\bar{k}}{1 - \bar{q}(1 + \bar{\beta}\bar{f})} < \Delta_m
\]  

\[20.\]  

\[21.\]  

\[22.\]  

\[23.\]  

\[24.\]  

\[25.\]
both hold, we have the ordering:

$$P^{nd} - \Delta_m < P^{nd} - \frac{\tilde{q}k}{1-q(1+\beta f)} < P^{nd}. \tag{25}$$

In this case (where the parameters are such that (22) and (24) both hold), we see that the least upper bound of the set $ND^m$ of materially withheld information, $\min\{P^{nd} - \frac{\tilde{q}k}{1-q(1+\beta f)}, P^{nd} - \Delta_m\}$ (see (23)), is $P^{nd} - \Delta_m$. Thus, the union of all withheld information, material or not, $ND = ND^{im} \cup ND^m$, is the set:

$$ND = \{v | v < P^{nd}\}. \tag{26}$$

In this case, in view of (26), the no disclosure price $P^{nd}$ serves as both the least upper bound of the no disclosure set and the expected value of the firm given no disclosure, just as in the analysis in the previous sections of the paper. Consequently, the characterization of the equilibrium no disclosure price and the equilibrium no disclosure set is exactly as it was previously, i.e., it is the fixed point $v^c$ of the function $EVF(v^c)$ (recall (9) above), so - in this case - shifting the assignemnt of liability from the shareholders of record at time 3 to the shareholders of record at time 2 has no effect on either the firm’s equilibrium disclosure policy or the firm’s equilibrium market value given no disclosure.

But this conclusion is not parameter-free. Suppose (22) continues to hold, but inequality (24) is reversed. Then, rather than having the order (25), we have the ordering:

$$P^{nd} - \frac{\tilde{q}k}{1-q(1+\beta f)} < P^{nd} - \Delta_m < P^{nd}. \tag{27}$$

In this case, the equilibrium no disclosure set is no longer connected: in view of (21) and (19), it consists of the union of two separate intervals $ND^m = (-\infty, P^{nd} - \frac{\tilde{q}k}{1-q(1+\beta f)})$ and $ND^{im} = (P^{nd} - \Delta_m, P^{nd})$. What happens in this case is that the absence of liability resulting from withholding immaterial information provides the firm with a safe harbor that the firm exploits to the "max:" any immaterial information that it receives that is below what it conjectures to be the equilibrium no disclosure price it withholds, and it discloses some worse information - $v'$s below $P^{nd} - \Delta_m$ - that it receives because of the fear of being subjected to liability if the fact finder discovers it has withheld that information. For very bad information - $v'$s below $P^{nd} - \frac{\tilde{q}k}{1-q(1+\beta f)}$: it still prefers to withhold, however. While the equilibrium no disclosure set is still determined by the equilibrium no disclosure price $P^{nd}$, neither that set nor the no disclosure price have the same characterization as they did in previous sections of the paper, since the no disclosure set is no longer given by the interval $[v, P^{nd})$.

Finally, we consider what happens when parameters change so that inequality (22) is reversed. This reversal has no effect on the description of the set of $v'$s that the firm decides to withhold that are immaterial: that set is still given as in (19) above. But, it does change the description of the set of $v'$s that the firm decides to withhold that are material: that set is now given by:

$$ND^m = \{v | v > P^{nd} - \frac{\tilde{q}k}{1-q(1+\beta f)} \} \cap \{v | v \leq P^{nd} - \Delta_m\}.$$  

But notice that this last set is empty, because $P^{nd} - \frac{\tilde{q}k}{1-q(1+\beta f)} > P^{nd}$ when inequality (22) is reversed. Thus, when inequality (22) is reversed, no MI that is received is withheld, and the (entire) no disclosure set
is given by the interval \( ND = (P_{nd} - \Delta_m, P_{nd}) \). The MT again serves as a safe haven to allow the firm to withhold some information it receives, but no "bad" information (below the no disclosure price \( P_{nd} \)) that is material is withheld. While the equilibrium no disclosure price serves to delimit the boundaries of the no disclosure set here, as in the previous case discussed above too, the characterization of the equilibrium no disclosure price and the no disclosure set are distinct from that provided in previous sections of the paper, since the equilibrium region is now the interval \( (P_{nd} - \Delta_m, P_{nd}) \) rather than \([v, P_{nd}]\).

The equilibrium "no disclosure" policy \( ND = (P_{nd} - \Delta_m, P_{nd}) \) that emerges from this last parameterization is consistent with Skinner's [1994], [1997] results that shows that legal considerations, combined with firms' usual incentives to disclose good news, can motivate firms to disclose voluntarily both sufficiently good and sufficiently bad news.\(^{30}\)

7 Conclusions

This paper extends the existing analytical literature on voluntary disclosures by examining an expected value-maximizing firm's disclosure decision in a model where the firm is obliged to make damage payments in the event it is determined to have withheld material information in its possession from investors. The specification of the firm's damage payments has several "real world" features: the damages are based on the difference between the "artificially high" market value of the firm while the information is withheld and the market value of the firm that would have prevailed had the firm disclosed its information; the damages increase with the fraction of the firm's shared that are purchased during the time the firm's shares were traded at the "artificially high" price; the damages are assessed only when the firm is deemed to have withheld MI; the damages are assessed on the firm/the firm's shareholders after some fact finder has determined that the firm previously withheld MI; the damages are affected by a "damages multiplier" that determines the extent of reimbursement that "wronged" investors (investors who purchased the firm's shares at the "artificially high" price) receive; and the damage payments are accompanied by litigation costs that have both fixed and variable cost components. The model in the paper further extends the existing disclosure literature by recognizing that there may be a distinction between the information that resides somewhere inside a firm and the information that the firm's senior managers are aware the firm has received. Since the probability that a firm's senior managers are aware of information that resides somewhere inside their organization is affected by the firm's organizational design (e.g., senior managers of centralized firms are more likely to be aware of the arrival of such information than senior managers of decentralized firms), the paper provides a rudimentary step in the development of a theory of the dependence of a firm's voluntary disclosure policy on its organizational design.

The paper describes a firm's equilibrium voluntary disclosure policy both in the case where the firm's investment level is exogenous and in the case where the firm's investment level is endogenous. An extensive set of comparative statics involving the dependence of the firm's equilibrium voluntary disclosure policy, the firm's equilibrium investment level, and the firm’s equilibrium expected market value on a variety of the model's parameters (including: the probability the firm receives information, the probability that senior

\(^{30}\)Wagenhofer [1990] contains another of voluntary disclosure where the set of disclosed information is not connected, based on very different reasoning from the present model.
managers are aware that the firm has received information, the MT, the size of the damages multiplier, the fraction of investors who purchase the firm’s shares at the "artificially high" price, the probability a fact finder discovers the firm previously withheld MI, the fixed and variable legal costs attending the damage payments) is obtained.

The paper also examines some of the consequences of reassigning responsibility for making damage payments from the firm/shareholders of record after a fact finder discovers that the firm previously withheld MI to the firm/shareholders of record at the time the firm makes its disclosure decision. The paper shows that while, for some natural parameter specifications, this reassignment of responsibility for making damage payments has no impact on the form or characterization of the firm’s equilibrium voluntary disclosure policy, there are other equally natural parameter specifications for which this reassignment of liability has a dramatic influence on the form of the firm’s equilibrium disclosure policy. Specifically, even though under the conventional assignment of responsibility, a firm’s equilibrium disclosure policy is always defined by a cutoff (with favorable information above the cutoff disclosed and unfavorable information below the cutoff withheld), with the reassignment of responsibility for making damage payments, the firm’s equilibrium disclosure policy can have large "gaps" in terms of what information the firm receives that it discloses: for some reasonable parameterizations, very unfavorable information is withheld; somewhat unfavorable information is disclosed; modestly unfavorable information is withheld; and favorable information is disclosed.

8 References


### 9 Appendix

**Proof of Lemma 1**

The equation $v^c = EFV(v^c)$ is:

$$v^c = \frac{pqG(v^c - \Delta_m)}{1 - p + p(1 - r + rG(v^c))} \times E[\tilde{v} + (1 - \tilde{f}) \times \beta \times (P_{1}^{ND} - \tilde{v}) - \tilde{k}|\tilde{v} < v^c - \tilde{\Delta}_m] +$$
Multiply both sides of this equation by $1 - p + p(1 - r + rG(v^c))$ to get:

$$v^c(1 - p + p(1 - r + rG(v^c))) = pq(G(v^e) - G(v^e - \Delta_m)) \times E[\tilde{v}]v^e - \tilde{\Delta}_m < \tilde{v} < v^e] +$$

$$pq(1 - G(v^c))(1 - r) \times E[\tilde{v}]v^e < \tilde{v}] +$$

$$(1 - p + p(1 - r + rG(v^c))r + 1 - r) \times$$

$$\{rG(v^e)p(1 - \tilde{q})E[\tilde{v}]\tilde{v} < v^e] + (1 - r) \times$$

$$p(1 - G(v^c))(1 - r)\tilde{q} \times E[\tilde{v}]v^e < \tilde{v}] +$$

$$(1 - p + p(1 - r + rG(v^c)) - pq(G(v^e)r + 1 - r)) \times$$

$$\{rG(v^e)p(1 - \tilde{q}) \times E[\tilde{v}]\tilde{v} < v^e] + (1 - p + p(rG(v^e) + 1 - r)(1 - \tilde{q})) \times E[\tilde{v}].$$

Rearranging terms and recognizing that, in equilibrium, $P^{ND} = v^e$, we get for the RHS(28):

$$pq(G(v^e) - \Delta_m) \times \{(1 - 1 - f)\beta E[\tilde{v}]\tilde{v} < v^e - \tilde{\Delta}_m] + (1 - f)\beta v^e - \tilde{k}\} +$$

$$pq(G(v^e) - G(v^e - \Delta_m)) \times E[\tilde{v}]v^e - \tilde{\Delta}_m < \tilde{v} < v^e] +$$

$$p(1 - G(v^e))(1 - r)\tilde{q} \times E[\tilde{v}]v^e < \tilde{v}] +$$

$$(1 - p + p(1 - q)1 - r + rG(v^e)) \times$$

$$\{rG(v^e)p(1 - \tilde{q})E[\tilde{v}]\tilde{v} < v^e] + (1 - p + p(rG(v^e) + 1 - r)(1 - \tilde{q})) \times E[\tilde{v}].$$

From this point of the proof onward, all instances of the random variables $\tilde{q}, \tilde{\beta}, \tilde{k}$ and $\tilde{f}$ in this equation only appear in terms of their respective means $\tilde{q}, \tilde{\beta}, \tilde{k},$ and $\tilde{f}$. Since the expressions that follow appear somewhat cleaner when we write, for example, $q$ in place of $\tilde{q}$, in the following, we shall eliminate the diacriticals from each of $\tilde{q}, \tilde{\beta}, \tilde{k},$ and $\tilde{f}$. Continuing our rearrangements with this new notation, this last expression can be rewritten:

$$pq(G(v^e) - \Delta_m) \times \{(1 - (1 - f)\beta)E[\tilde{v}]\tilde{v} < v^e - \Delta_m] + (1 - f)\beta v^e - k\} +$$

$$pq(G(v^e) - G(v^e - \Delta_m)) \times E[\tilde{v}]v^e - \Delta_m < \tilde{v} < v^e] +$$

$$p(1 - G(v^e))(1 - r)q \times E[\tilde{v}]v^e < \tilde{v}] +$$

$$\{rG(v^e)p(1 - q)E[\tilde{v}]\tilde{v} < v^e] + (1 - p + p(1 - q)(1 - r + rG(v^e)) - rG(v^e)p(1 - q)) \times E[\tilde{v}].$$

or as

$$pq(G(v^e) - \Delta_m) \times \{(1 - (1 - f)\beta)E[\tilde{v}]\tilde{v} < v^e - \Delta_m] + (1 - f)\beta v^e - k\} +$$

$$pq(G(v^e) - G(v^e - \Delta_m)) \times E[\tilde{v}]v^e - \Delta_m < \tilde{v} < v^e] +$$

$$p(1 - G(v^e))(1 - r)q \times E[\tilde{v}]v^e < \tilde{v}] +$$

$$\{rG(v^e)p(1 - q)E[\tilde{v}]\tilde{v} < v^e] + (1 - p + p(1 - q)(1 - r + rG(v^e)) - rG(v^e)p(1 - q)) \times E[\tilde{v}].$$
$$pq(G(v^c) - G(v^c - \Delta_m)) \times E[\hat{v}]v^c < \hat{v} < v^c] +$$

$$pq(1 - G(v^c))(1 - r) \times E[\hat{v}]v^c < \hat{v} [ ] +$$

$$rG(v^c)p(1 - q)E[\hat{v}]v^c < \hat{v} [ ] + (1 - p + p(1 - q)(1 - r)) \times E[\hat{v}]$$

or as

$$pq \times \{(1 - (1 - f)\beta)\int_{v^c}^{v^c - \Delta_m} vg(v)dv + ((1 - f)\beta v^c - k)G(v^c - \Delta_m)\} +$$

$$pq \times \int_{v^c}^{v^c} vg(v)dv +$$

$$p(1 - r)q \times \int_{v^c}^{v^c} vg(v)dv +$$

$$rp(1 - q)\int_{v^c}^{v^c} vg(v)dv + (1 - p + p(1 - q)(1 - r)) \times E[\hat{v}]$$

or as

$$pq \times \{(1 - (1 - f)\beta)((v^c - \Delta_m)G(v^c - \Delta_m) - \int_{v^c}^{v^c - \Delta_m} G(v)dv) +$$

$$+((1 - f)\beta v^c - k)G(v^c - \Delta_m)\} +$$

$$pq \times \{v^cG(v^c) - (v^c - \Delta_m)G(v^c - \Delta_m) - \int_{v^c - \Delta_m}^{v^c} G(v)dv\} +$$

$$p(1 - r)q \times \{\hat{v} - v^cG(v^c) - \int_{v^c}^{\hat{v}} G(v)dv\} +$$

$$rp(1 - q)\times \{v^cG(v^c) - \int_{v^c}^{\hat{v}} G(v)dv + (1 - p + p(1 - q)(1 - r)) \times E[\hat{v}]$$

or as

$$pq \times \{(v^c - \Delta_m) + (1 - f)\beta \Delta_m - k)G(v^c - \Delta_m) - (1 - (1 - f)\beta)\int_{v^c}^{v^c - \Delta_m} G(v)dv\} +$$

$$pq \times \{v^cG(v^c) - (v^c - \Delta_m)G(v^c - \Delta_m) - \int_{v^c - \Delta_m}^{v^c} G(v)dv\} +$$

$$p(1 - r)q \times \{\hat{v} - v^cG(v^c) - \int_{v^c}^{\hat{v}} G(v)dv\} +$$

$$rp(1 - q)\times \{v^cG(v^c) - \int_{v^c}^{\hat{v}} G(v)dv + (1 - p + p(1 - q)(1 - r)) \times E[\hat{v}]$$

or as

$$pq \times \{((1 - f)\beta \Delta_m - k)G(v^c - \Delta_m) - (1 - (1 - f)\beta)\int_{v^c}^{v^c - \Delta_m} G(v)dv\} +$$

$$-pq \int_{v^c - \Delta_m}^{v^c} G(v)dv + p(1 - r)q \times \{\hat{v} - \int_{v^c}^{\hat{v}} G(v)dv\} +$$

$$prv^cG(v^c) - rp(1 - q)\int_{v^c}^{v^c} G(v)dv + (1 - p + p(1 - q)(1 - r)) \times E[\hat{v}]$$

or as

$$pq((1 - f)\beta \Delta_m - k)G(v^c - \Delta_m) + pq(1 - f)\beta \int_{v^c}^{v^c - \Delta_m} G(v)dv +$$

32
\[-p(r + q(1 - r)) \int_{v}^{e} G(v)dv + p(1 - r)q \times \{\bar{v} - \int_{v}^{e} G(v)dv\} + \]

\[prv\beta G(v^\beta) + (1 - p + p(1 - q)(1 - r)) \times E[\bar{v}]\]

or as

\[pq((1 - f)\beta \Delta_m - k)G(v - \Delta_m) + pq(1 - f)\beta \int_{v}^{e}G(v)dv + \]

\[-pr \int_{v}^{e} G(v)dv + p(1 - r)q \times \{\bar{v} - \int_{v}^{e} G(v)dv\} + \]

\[prv\beta G(v^\beta) + (1 - p + p(1 - q)(1 - r)) \times E[\bar{v}]\]

or as

\[pq((1 - f)\beta \Delta_m - k)G(v - \Delta_m) + pq(1 - f)\beta \int_{v}^{e}G(v)dv + \]

\[-pr \int_{v}^{e} G(v)dv + p(1 - r)qE[\bar{v}] + \]

\[prv\beta G(v^\beta) + (1 - p + p(1 - q)(1 - r)) \times E[\bar{v}]\]

or as

\[pq((1 - f)\beta \Delta_m - k)G(v - \Delta_m) + pq(1 - f)\beta \int_{v}^{e}G(v)dv + \]

\[-pr \int_{v}^{e} G(v)dv + prv\beta G(v^\beta) + (1 - pr) \times E[\bar{v}]\].

Therefore, after deleting \(prv\beta G(v^\beta)\) from both sides of the equation determined by equating this last expression with \(LHS(28)\), and then dividing this equation by \(1 - pr\), we conclude that equation (28) is equivalent to (10) in the text. \(\blacksquare\)

**Proof of Lemma 2**

We start by proving the sufficiency of condition (a) for the existence of a unique equilibrium by making four observations.

**Observation 1:** \(LHS(10)\) is strictly greater than \(RHS(10)\) when \(v^\epsilon = \bar{v}\). (This is true under both conditions (a) and (b)). Clearly, \(LHS(10)\) is positive when \(v^\epsilon = \bar{v}\) and \(RHS(10)\) is zero when \(v^\epsilon = \underline{v}\).

**Observation 2:** \(LHS(10)\) is strictly smaller than \(RHS(10)\) when \(v^\epsilon = \bar{v}\). At \(v^\epsilon = \bar{v}\), \(LHS(10)\) is strictly negative, whereas when \(\bar{f} \geq 1\), \(RHS(10)\) is obviously positive for any \(v^\epsilon > \bar{v}\), including \(v^\epsilon = \bar{v}\). This proves Observation 2 under condition (a).

**Observation 3:** There exists at least one \(v^\epsilon \in [\underline{v}, \bar{v}]\) at which \(LHS(10) = RHS(10)\). This follows from Observations 1 and 2, the fact that both sides of \((10)\) are continuous in \(v^\epsilon\), and the intermediate value theorem.

**Observation 4:** There is exactly one \(v^\epsilon \in [\underline{v}, \bar{v}]\) satisfying \(LHS(10) = RHS(10)\). Take the derivative with respect to \(v^\epsilon\) of \(LHS(10) - RHS(10)\). This derivative is given by:

\[-1 - \frac{pr}{1 - pr} G(v^\epsilon) + \frac{pq}{1 - pr} \times [((1 - \bar{f})\beta \Delta_m - k)g(v^\epsilon - \Delta_m) + (1 - \bar{f})\beta G(v^\epsilon - \Delta_m)] \quad (29)\]

When \(\bar{f} \geq 1\), clearly (29) is negative.
This proves the uniqueness of the \( v^c \in (\bar{v}, \bar{v}) \) that solves (10), as it shows that the difference \( LHS(10) - RHS(10) \), which starts out positive at \( v^c = \bar{v} \) by Observation 1 and winds up negative at \( v^c = \bar{v} \) by Observation 2, and is strictly decreasing for all \( v^c \in (\bar{v}, \bar{v}) \) by Observation 3.

Next, we prove the lemma under condition (b). We already noted that Observation 1 holds under both conditions (a) and (b), and that \( LHS(10) \) evaluated at \( v^c = \bar{v} \) is negative. To prove Observation 2 under condition (b), consider the maximization problem:

\[
\max_{\Delta_m \geq 0} \vartheta(\Delta_m), \tag{30}
\]

where \( \vartheta(\Delta_m) \equiv ((1 - \bar{f})\bar{\beta}\Delta_m - \bar{k})G(\bar{v} - \Delta_m) + (1 - \bar{f})\bar{\beta} \int_{\bar{v}}^{\bar{v} - \Delta_m} G(v)dv \). Either the maximum of this program is achieved by \( \Delta_m = 0 \) or not. If it is, then notice that when \( r > \bar{q}(1 - \bar{f})\bar{\beta} \), \( RHS(10) \) evaluated at \( v^c = \bar{v} \) is given by:

\[
\frac{pr}{1 - pr} \bar{\beta} \int_{\bar{v}}^{\bar{v}} G(v)dv - \frac{pq}{1 - pr} \bar{\beta} \int_{\bar{v}}^{\bar{v} - \Delta_m} G(v)dv = \vartheta(\Delta_m)
\]

\[
\geq \frac{pr}{1 - pr} \bar{\beta} \int_{\bar{v}}^{\bar{v}} G(v)dv - \frac{pq}{1 - pr} \bar{\beta} \int_{\bar{v}}^{\bar{v}} G(v)dv = \vartheta(0)
\]

\[
= \frac{pr}{1 - pr} \bar{\beta} \int_{\bar{v}}^{\bar{v} - \Delta_m} G(v)dv + (1 - \bar{f})\bar{\beta} \int_{\bar{v}}^{\bar{v} - \Delta_m} G(v)dv = 0
\]

(the inequality (31) follows from the present assumption that the program (30) is achieved by \( \Delta_m = 0 \), so Observation 2 holds.

Alternatively, if the maximum of the program in (30) is not achieved by \( \Delta_m = 0 \), then the maximizing value of \( \Delta_m \) satisfies the first-order condition:

\[
-(1 - \bar{f})\bar{\beta}\Delta_m - \bar{k}g(\bar{v} - \Delta_m) = 0. \tag{32}
\]

Obviously, with \( \Delta_m^* \equiv \frac{k}{(1 - \bar{f})\bar{\beta}} \), precisely one of: (b1) \( \bar{v} - \bar{v} > \Delta_m^* \) or (b2) \( \bar{v} - \bar{v} \leq \Delta_m^* \) holds.

In case (b1), we have \( \bar{v} - \Delta_m^* \in (\bar{v}, \bar{v}) \). Since \( g(\cdot) \) is positive on \( (\bar{v}, \bar{v}) \), \( \frac{31}{1} \) it follows that the solution to the first-order condition (32) is: \( \Delta_m^* \equiv \frac{k}{(1 - \bar{f})\bar{\beta}} \). In this case, the second-order condition for a maximum is satisfied at \( \Delta_m^* = \frac{k}{(1 - \bar{f})\bar{\beta}} \) when \( \bar{f} < 1 \), since the second-order derivative of LHS(32) is given by \( -(1 - \bar{f})\bar{\beta}g(\bar{v} - \Delta_m^*) < 0. \tag{32} \)

Thus, evaluating the program (30) at its optimum, \( \vartheta(\Delta_m^*) = (1 - \bar{f})\bar{\beta} \int_{\bar{v}}^{\bar{v} - \Delta_m^*} G(v)dv \). So, when

\[31\] By definition of \( [\bar{v}, \bar{v}] \) being the support of \( \bar{v} \).

\[32\] Written out, the second derivative with respect to \( \Delta_m \) is given by:

\[
-(1 - \bar{f})\bar{\beta}g(\bar{v} - \Delta_m^*) + ((1 - \bar{f})\bar{\beta}\Delta_m^* - \bar{k})g'(\bar{v} - \Delta_m)
\]

\[
= -(1 - \bar{f})\bar{\beta}g(\bar{v} - \Delta_m^*),
\]

where the equality comes from substituting from the first-order condition.
\( r > \tilde{q}(1 - \tilde{f})\tilde{\beta} \), a lower bound on \( \text{RHS}(10) \) evaluated at \( v^c = \tilde{v} \) is given by (taking into account the definition of the program (30)):

\[
\frac{pr}{1 - pr} \int_{\varepsilon}^{\tilde{v}} G(v)dv - \frac{pq}{1 - pr} \times (1 - \tilde{f})\tilde{\beta} \int_{\varepsilon}^{\tilde{v} - \frac{k}{(1-\tilde{f})\tilde{\beta}}} G(v)dv \\
\geq \frac{pr}{1 - pr} \left( \int_{\varepsilon}^{\tilde{v}} G(v)dv - \int_{\tilde{v} - \frac{k}{(1-\tilde{f})\tilde{\beta}}}^{\tilde{v}} G(v)dv \right) \\
= \frac{pr}{1 - pr} \int_{\varepsilon}^{\tilde{v} - \frac{k}{(1-\tilde{f})\tilde{\beta}}} G(v)dv > 0. \tag{33}
\]

Thus, Observation 2 holds in case (b1) too.

In case (b2), where \( \tilde{v} - \tilde{\nu} \leq \Delta_m = \frac{k}{(1-\tilde{f})\tilde{\beta}} \), begin by noting that if \( \Delta_m < \tilde{v} - \tilde{\nu} \), then \( \Delta_m < \frac{k}{(1-\tilde{f})\tilde{\beta}} \) and \( \tilde{\nu} < \tilde{v} - \Delta_m \) both hold, and hence the following two inequalities also hold: \( (1 - \tilde{f})\tilde{\beta}\Delta_m - \tilde{k} < 0 \) and \( g(\tilde{v} - \Delta_m) > 0 \) (the last inequality holds since \( \tilde{v} - \Delta_m \) is in the support of \( \tilde{v} \)). Hence, \( \text{LHS}(32) > 0 \) for any \( \Delta_m < \tilde{v} - \tilde{\nu} \). This implies that the objective function in the maximization problem (30) is increasing in \( \Delta_m \) for all \( \Delta_m \leq \tilde{v} - \tilde{\nu} \). Since the objective function is constant for \( \Delta_m > \tilde{v} - \tilde{\nu} \) (this is clear from (32), since \( \text{LHS}(32) = 0 \) for any \( \Delta_m > \tilde{v} - \tilde{\nu} \)), it follows that the maximum value of the program (30) is achieved at \( \Delta_m = \tilde{v} - \tilde{\nu} \). Since \( \vartheta(\tilde{v} - \tilde{\nu}) = 0 \), it follows that, in case (b2) also, Observation 2 holds, since by the same logic as used to deduce (31) above, we conclude \( \text{RHS}(10) \) evaluated at \( v^c = \tilde{v} \) is given by:

\[
\frac{pr}{1 - pr} \int_{\varepsilon}^{\tilde{v}} G(v)dv - \frac{pq}{1 - pr} \times \vartheta(\Delta_m) \\
\geq \frac{pr}{1 - pr} \int_{\varepsilon}^{\tilde{v}} G(v)dv - \frac{pq}{1 - pr} \times \vartheta(\tilde{v} - \tilde{\nu}) \\
= \frac{pr}{1 - pr} \int_{\varepsilon}^{\tilde{v}} G(v)dv > 0.
\]

Observation 3 is immediate. To prove Observation 4 under condition (b), note that since \( \tilde{k} \geq 0 \) and \( r > \tilde{q}(1 - \tilde{f})\tilde{\beta} \), the derivative (29) is at least weakly smaller than

\[
-1 - \frac{pr}{1 - pr} G(v^c) + \frac{pq(1 - \tilde{f})\tilde{\beta}}{1 - pr} \times [\Delta_m g(v^c - \Delta_m) + G(v^c - \Delta_m)] \\
\leq -1 - \frac{pr}{1 - pr} (G(v^c) - \Delta_m g(v^c - \Delta_m) - G(v^c - \Delta_m)) \\
= -1 - \frac{pr\Delta_m}{1 - pr} (g(\xi) - g(v^c - \Delta_m)), \tag{34}
\]

where the equality follows because of the intermediate value theorem for some \( \xi \in [v^c - \Delta_m, v^c] \). Recalling that \( g(\cdot) \) is bounded by .5\( \rho \), it follows that (34) is no bigger than \( -1 + \frac{pr\Delta_m\tilde{k}}{1 - pr} \). Hence, as long as \( \Delta_m < \frac{1 - pr}{pr\tilde{k}} \), then (34) is negative, in which case, by the same reasoning used to prove Observation 4 under condition (a) above, it follows that the solution to (10) is unique. 

**Proof of Theorem 1**

When the equilibrium value for the cutoff \( v^c \) is considered to be a function of the model’s parameters, i.e., \( v^c = v^c(\Delta_m, \tilde{f}, \tilde{k}, \tilde{\beta}, \tilde{q}, p) \), then equation (10) is an identity in those parameters. The identity is preserved upon total differentiation with respect to any of these parameters. An easy way to differentiate (10) totally
with respect to any parameter \( \gamma \in \{ \Delta_m, \tilde{f}, \tilde{k}, \tilde{\beta}, \tilde{q}, p \} \) is to first differentiate it with respect to \( v^c \), i.e., to calculate:

\[
\frac{\partial \text{LHS}(10)}{\partial v^c} \times \frac{dv^c}{d\gamma} + \frac{\partial \text{LHS}(10)}{\partial \gamma} = \frac{\partial \text{RHS}(10)}{\partial v^c} \times \frac{dv^c}{d\gamma} + \frac{\partial \text{RHS}(10)}{\partial \gamma}.
\]

Since \( \frac{\partial \text{LHS}(10)}{\partial v^c} = -1, \frac{\partial \text{LHS}(10)}{\partial \gamma} = 0 \), and

\[
\frac{\partial \text{RHS}(10)}{\partial v^c} = D(v^c) \equiv \frac{p}{1 - pr} \left[ rG(v^c) - \tilde{q} \left\{ (1 - \bar{f})\tilde{\beta}\Delta_m - \bar{k} \right\} g(v^c - \Delta_m) + (1 - \bar{f})\tilde{\beta}G(v^c - \Delta_m) \right],
\]

it follows that

\[
\frac{dv^c}{d\gamma} = \frac{\frac{\partial \text{RHS}(10)}{\partial v^c}}{1 + D(v^c)}. \tag{35}
\]

Comparative statics then obtain from evaluating (35) for the various parameters \( \gamma \in \{ \Delta_m, \tilde{f}, \tilde{\beta}, \tilde{q}, p \} \).

To proceed, we first must sign the denominator \( 1 + D(v^c) \) of \( \frac{dv^c}{d\gamma} \) in (35). We have the following result.

**Lemma 4** Under either of the conditions of Lemma 2, \( 1 + D(v^c) > 0 \).

**Proof.** When \( \bar{f} \geq 1 \), the claim is obvious, since \( D(v^c) \) is weakly bigger than

\[
\frac{p}{1 - pr} rG(v^c) - \tilde{q}(1 - \bar{f})\tilde{\beta}(\Delta_m g(v^c - \Delta_m) + G(v^c - \Delta_m)),
\]

and the latter is obviously positive if \( \bar{f} \geq 1 \).

When \( r > \tilde{q}(1 - \bar{f})\tilde{\beta} \):

\[
\begin{align*}
rG(v^c) - \tilde{q} \left\{ (1 - \bar{f})\tilde{\beta}\Delta_m - \bar{k} \right\} g(v^c - \Delta_m) + (1 - \bar{f})\tilde{\beta}G(v^c - \Delta_m) \\
\geq rG(v^c) - \tilde{q}(1 - \bar{f})\tilde{\beta} \{ \Delta_m g(v^c - \Delta_m) + G(v^c - \Delta_m) \} \\
\geq r(G(v^c) - G(v^c - \Delta_m) - \Delta_m g(v^c - \Delta_m)) \\
= r\Delta_m (g(\xi) - g(v^c - \Delta_m)) \\
\geq -r\Delta_m b,
\end{align*}
\]

(where the equality follows for some \( \xi \in [v^c - \Delta_m, v^c] \) (by the mean value theorem) and the last inequality follows from \( g(\cdot) \) being bounded by \( .5b \). It follows that

\[
rG(v^c) - \tilde{q} \left\{ (1 - \bar{f})\tilde{\beta}\Delta_m - \bar{k} \right\} g(v^c - \Delta_m) + (1 - \bar{f})\tilde{\beta}G(v^c - \Delta_m) \geq -r\Delta_m b.
\]

Therefore, \( 1 + D(v^c) > 0 \) surely holds when \( 1 - \frac{pr\Delta_m}{1 - pr} > 0 \). A little algebra shows that \( 1 - \frac{pr\Delta_m}{1 - pr} > 0 \) iff inequality \( \Delta_m < \frac{1 - pr}{prb} \), and the latter inequality is among the conditions stipulated in Lemma 2(b). \( \blacksquare \)

When \( 1 + D(v^c) > 0 \), we conclude by (35) that for any \( \gamma \in \{ \Delta_m, \tilde{f}, \tilde{\beta}, \tilde{q}, p \} \):

\[
\text{sgn} \frac{dv^c}{d\gamma} = -\text{sgn} \left( \frac{\partial \text{RHS}(10)}{\partial \gamma} \right). \tag{36}
\]

This last observation is the basis for the proofs of most parts of Theorem 1.

**Proof of Theorem 1(a)**

Applying (36) to \( \gamma = \Delta_m \), we conclude:

\[
\text{sgn} \frac{dv^c}{d\Delta_m} = -\text{sgn} \left( \frac{\partial \text{RHS}(10)}{\partial \Delta_m} \right)
\times \frac{-pg \times ((1 - \bar{f})\tilde{\beta}\Delta_m - \bar{k}) \times g(v^c - \Delta_m)}{1 - pr}
\times -\text{sgn}(1 - \bar{f})\tilde{\beta}\Delta_m - \bar{k}.
\]

\[
\tag{37}
\text{sgn} \frac{dv^c}{d\Delta_m} = -\text{sgn}(1 - \bar{f})\tilde{\beta}\Delta_m - \bar{k}.
\]

\[
\tag{38}
\]
Thus, $\text{sgn} \frac{dv^e}{d\alpha}$ is negative or positive, depending on whether $(1 - \bar{f})\bar{\beta}\Delta_m - \bar{k} > 0$ or $(1 - \bar{f})\bar{\beta}\Delta_m - \bar{k} < 0$.

**Proof of Theorem 1(b)**

Applying (36) when $\gamma = \bar{\beta}$, we conclude:

$$\text{sgn} \frac{dv^e}{d\beta} = \text{sgn} \left( (1 - \bar{f}) \left[ \Delta_m G(v^e - \Delta_m) + \int_{v^e - \Delta_m}^{v^e} G(v) dv \right] \right)$$

$$= \text{sgn}(1 - \bar{f}).$$

**Proof of Theorem 1(c)**

Applying (36) when $\gamma = \bar{q}$, we conclude:

$$\text{sgn} \frac{dv^e}{dq} = \text{sgn} \left[ ((1 - \bar{f})\bar{\beta}\Delta_m - \bar{k})G(v^e - \Delta_m) + (1 - \bar{f})\bar{\beta} \int_{v^e - \Delta_m}^{v^e} G(v) dv \right],$$

and the latter is positive when $(1 - \bar{f})\bar{\beta}\Delta_m > \bar{k}$ and is negative when $1 - \bar{f} < 0$.

**Proof of Theorem 1(d)**

Applying (36) when $\gamma = \bar{f}$, we conclude:

$$\text{sgn} \frac{dv^e}{df} = -\text{sgn} \left( \bar{\beta}\Delta_m G(v^e - \Delta_m) + \bar{\beta} \int_{v^e - \Delta_m}^{v^e} G(v) dv \right) < 0.$$

**Proof of Theorem 1(e)**

Applying (36) when $\gamma = \bar{k}$, we conclude:

$$\text{sgn} \frac{dv^e}{dk} = -\text{sgn} \left( \frac{pq}{1 - pr} \times G(v^e - \Delta_m) \right) < 0.$$

**Proof of Theorem 1(f)**

This follows from Theorem 1(d) and the convention (4).

**Proof of Theorem 1(g)**

Applying (36) when $\gamma = p$, we conclude

$$\text{sgn} \frac{dv^e}{dp} = -\text{sgn} \left( r \int_{v^e}^{\infty} G(v) dv - \bar{q} \times \left[ ((1 - \bar{f})\bar{\beta}\Delta_m - \bar{k})G(v^e - \Delta_m) + (1 - \bar{f})\bar{\beta} \int_{v^e - \Delta_m}^{v^e} G(v) dv \right] \right)$$

It follows that $\text{sgn} \frac{dv^e}{dp} < 0$, since

$$r \int_{v^e}^{\infty} G(v) dv - \bar{q} \times \left[ ((1 - \bar{f})\bar{\beta}\Delta_m - \bar{k})G(v^e - \Delta_m) + (1 - \bar{f})\bar{\beta} \int_{v^e - \Delta_m}^{v^e} G(v) dv \right]$$

$$\geq r \int_{v^e}^{\infty} G(v) dv - \bar{q} \times \left[ (1 - \bar{f})\bar{\beta}\Delta_m G(v^e - \Delta_m) + (1 - \bar{f})\bar{\beta} \int_{v^e - \Delta_m}^{v^e} G(v) dv \right]$$

$$> r \int_{v^e - \Delta_m}^{v^e} G(v) dv - r \times \left[ \Delta_m G(v^e - \Delta_m) + \int_{v^e - \Delta_m}^{v^e} G(v) dv \right]$$

$$\geq r \left[ \int_{v^e - \Delta_m}^{v^e} G(v) dv - \Delta_m G(v^e - \Delta_m) \right]$$

$$= r\Delta_m [G(\xi) - G(v^e - \Delta_m)]$$

$$> 0.$$
(The second to last line follows for some $\xi \in (v^e - \Delta_m, v^e)$ by the intermediate value theorem, and the last inequality follows from $G(\cdot)$, as a cdf of a random variable with support $[\underline{v}, \bar{v}]$, is strictly increasing on $[\underline{v}, \bar{v}]$.

**Proof of Theorem 1(h)**

Applying (36) when $\gamma = r$, we conclude:

$$sgn\frac{dv^e}{dr} = -sgn\left(\int_{\underline{v}}^{v^e} G(v)dv - \frac{pq}{1-pr} \times \left[\{(1 - \bar{f})\Delta_m - \bar{k}\}G(v^e - \Delta_m) + (1 - \bar{f})\bar{\beta} \int_{\underline{v}}^{v^e - \Delta_m} G(v)dv\right]\right)$$

Observe that, under the assumptions of the lemma, $sgn\frac{dv^e}{dr} < 0$ since $r > q(1 - \bar{f})\bar{\beta}$ implies $1 > q(1 - \bar{f})\bar{\beta}$ and so:

$$\int_{\underline{v}}^{v^e} G(v)dv - \frac{pq}{1-pr} \times \left[\{(1 - \bar{f})\Delta_m - \bar{k}\}G(v^e - \Delta_m) + (1 - \bar{f})\bar{\beta} \int_{\underline{v}}^{v^e - \Delta_m} G(v)dv\right]$$

$$\geq \int_{\underline{v}}^{v^e} G(v)dv - \frac{pq}{1-pr}(1 - \bar{f})\bar{\beta} \left[\Delta_m G(v^e - \Delta_m) + \int_{\underline{v}}^{v^e - \Delta_m} G(v)dv\right]$$

$$\geq \int_{\underline{v}}^{v^e} G(v)dv - \Delta_m G(v^e - \Delta_m) - \int_{\underline{v}}^{v^e - \Delta_m} G(v)dv$$

$$= \int_{\underline{v}}^{v^e - \Delta_m} G(v)dv - \Delta_m G(v^e - \Delta_m)$$

> 0,

(the last inequality follows from an argument like that used in (39) above).

**Proof of Theorem 1(i)**

To evaluate this, first we rewrite (10) by parametrizing $G(\cdot)$ by $I$, i.e.,

$$E_I[v] - v^e$$

$$= \frac{pr}{1 - pr} \int_{\underline{v}}^{v^e} G(v|I)dv - \frac{pq}{1 - pr} \times \left[\{(1 - \bar{f})\Delta_m - \bar{k}\}G(v^e - \Delta_m|I) + (1 - \bar{f})\bar{\beta} \int_{\underline{v}}^{v^e - \Delta_m} G(v|I)dv\right]$$

\[40\]

\[33\] It must be that $\xi \in (v^e - \Delta_m, v^e)$ and not $\xi = v^e - \Delta_m$ since $G(\cdot)$ is strictly increasing.
Write $E_1[\bar{v}]$ as $\int_{\underline{v}}^{\bar{v}} (1 - G(v|I)) dv$ and totally differentiate (40) with respect to $I$ to get:\(^{34}\)

$$\frac{dv^c}{dI} = - \int_{\underline{v}}^{\bar{v}} G_1(v|I) dv - \frac{pr}{1-pr} \int_{\underline{v}}^{\bar{v}} G_1(v|I) dv \cdot \frac{\partial G_1(v|I)}{\partial \Delta_m} \cdot \frac{d\bar{v}}{d\Delta_m}$$

\[

\geq - \int_{\underline{v}}^{\bar{v}} G_1(v|I) dv - \frac{pr}{1-pr} \int_{\underline{v}}^{\bar{v}} G_1(v|I) dv \cdot \frac{\partial G_1(v|I)}{\partial \Delta_m} \cdot \Delta_m \left( \frac{\partial G_1(v|I)}{\partial \Delta_m} - \frac{\partial G_1(v|I)}{\partial \Delta_m} \right) \\

\geq \frac{(\bar{v} - \underline{v}) B - \frac{pr \Delta_m}{1-pr} \left( G_1(\xi|I) - G_1(v^c - \Delta_m|I) \right)}{1 + D(v^c)},
\]

where we have used $|G_1| \leq -B$ for some $B > 0$, and $\psi$ and $\xi$ are some constants such that $\psi \in [\underline{v}, \bar{v}]$ and $\xi \in [v^c - \Delta_m, v^c]$. So, if $(\bar{v} - \underline{v}) \frac{1}{2pr} > \Delta_m$, then $\frac{dv^c}{dI} > 0.$ \(\blacksquare\)

**Proof of Lemma 3**

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**Proof of Theorem 2**

Initially, we hold all parameters other than $\Delta_m$ fixed. We write

$$I = I(\Delta_m) \text{ and } v^c = v^c(I(\Delta_m), \Delta_m)$$

(41)

for the equilibrium investment level and cutoff respectively, given the threshold $\Delta_m$.

Explicitly accounting for the dependence of $I$ and $v^c$ on $\Delta_m$, the first-order condition for $I$ is given by (recall (17) in the text):

$$N \equiv pr G_1(v^c(I(\Delta_m), \Delta_m)|I(\Delta_m)) v^c(I(\Delta_m), \Delta_m) + pr \int_{v^c(I(\Delta_m), \Delta_m)}^{\bar{v}} v g_1(v|I(\Delta_m)) dv - 1 = 0. \quad (42)$$

To obtain comparative statics regarding $\Delta_m$, we differentiate $N$ with respect to $\Delta_m$. This can be done as follows:

\[

\frac{dN}{d\Delta_m} = \frac{\partial N}{\partial v^c} \cdot \frac{\partial v^c(I(\Delta_m), \Delta_m)}{\partial I} \frac{dI(\Delta_m)}{d\Delta_m} + \frac{\partial N}{\partial v^c} \frac{dI(\Delta_m)}{d\Delta_m} + \frac{\partial N}{\partial v^c} \frac{d\bar{v}}{d\Delta_m} + \frac{\partial N}{\partial \Delta_m} \frac{dI(\Delta_m)}{d\Delta_m} + \frac{\partial N}{\partial \Delta_m} \frac{d\bar{v}}{d\Delta_m}.
\]

(43)

\(^{34}\) We write this differentiation out in full, rather than relying on the simplification provided by (36), because the full expression is useful for subsequent computations.
Substituting
\[
\frac{\partial N}{\partial v_c} = prG_I(v^c|I) \text{ (negative by FOSD)} \tag{44}
\]
\[
\frac{\partial v^c}{\partial I} > 0 \text{ (by Theorem 1(i))}
\]
\[
Q = \frac{\partial N}{\partial I} = prG_{II}(v^c|I)v^c + pr \int_{v_c}^{\bar{v}} v g_{II}(v|I)dv
\]  
(negative as the second order condition of (42); and
\[
\frac{\partial N}{\partial \Delta_m} = 0,
\]
we get (leaving out arguments to improve readability), with \( H \equiv \frac{\partial N}{\partial v_c} \times \frac{\partial v^c}{\partial I} < 0: \)
\[
(H + Q) \times \frac{dI(\Delta_m)}{d\Delta_m} + prG_I(v^c|I) \times \frac{\partial v^c}{\partial \Delta_m} = 0. \tag{46}
\]
Since \( \frac{\partial v^c}{\partial \Delta_m} \) is negative \((1 - \bar{f})\bar{\beta}\Delta_m - \bar{k} > 0 \) and positive when \((1 - \bar{f})\bar{\beta}\Delta_m - \bar{k} < 0 \) by Theorem 1(a), appealing to (44), we conclude that \( \frac{dI(\Delta_m)}{d\Delta_m} \) is negative when \((1 - \bar{f})\bar{\beta}\Delta_m - \bar{k} < 0 \) and is positive when \((1 - \bar{f})\bar{\beta}\Delta_m - \bar{k} >. \)

This same sequence of arguments can be used to obtain all the other comparative statics of Theorem 2. For example, consider \( \frac{dI(\bar{\beta})}{d\bar{\beta}} \). In this case, we write the first-order condition (42) as
\[
N = prG_I(v^c(I(\bar{\beta}), \bar{\beta}))v^c(I(\bar{\beta}), \bar{\beta}) + pr \int_{v_c(I(\bar{\beta}), \bar{\beta})}^{\bar{v}} v g_{I}(v|I(\bar{\beta}))dv - 1 = 0. \tag{47}
\]
Totally differentiating (47) with respect to \( \bar{\beta} \), we get, in analogy with (43):
\[
\frac{dN}{d\bar{\beta}} = \left( \frac{\partial N}{\partial v_c} \times \frac{\partial v^c(I(\bar{\beta}), \bar{\beta})}{\partial I} + \frac{\partial N}{\partial I} \right) \times \frac{dI(\bar{\beta})}{d\bar{\beta}} + \frac{\partial N}{\partial v_c} \times \frac{\partial v^c(I(\bar{\beta}), \bar{\beta})}{\partial \bar{\beta}} + \frac{\partial N}{\partial \bar{\beta}}
\]
\[
= \left( prG_I(v^c|I) \times \frac{\partial v^c}{\partial I} + prG_{II}(v^c|I)v^c + pr \int_{v_c}^{\bar{v}} v g_{II}(v|I)dv \right) \times \frac{dI(\bar{\beta})}{d\bar{\beta}} + prG_I(v^c|I) \times \frac{\partial v^c}{\partial \bar{\beta}} \tag{48}
\]
The coefficient of \( \frac{dI(\bar{\beta})}{d\bar{\beta}} \) in (48) is negative for the exact same reasons that the coefficient of \( \frac{dI(\Delta_m)}{d\Delta_m} \) in (46) was negative. Since \( \frac{\partial v^c}{\partial \bar{\beta}} \) is positive when \( \bar{f} < 1 \) and is negative when \( \bar{f} > 1 \) by Theorem 1(b), it follows that \( \frac{dI(\bar{\beta})}{d\bar{\beta}} \) is negative when \( \bar{f} < 1 \) and is positive when \( \bar{f} > 1 \).

The comparative statics results involving \( \frac{dI(\gamma)}{d\gamma} \) are computed similarly for all other \( \gamma \in \{ \bar{f}, \bar{\beta}, \bar{q}, p, r \}. \)

\[\text{[35]}\]
Proof of Theorem 3

By the envelope theorem, using the notation developed in (16) and (41) above:

\[
\frac{d}{d\Delta_m} W(v^c(I(\Delta_m), \Delta_m), I(\Delta_m))
\]

\[
= \frac{\partial}{\partial v^c} W(v^c(I(\Delta_m), \Delta_m), I(\Delta_m)) \times \\
\left\{ \frac{\partial v^c dI(\Delta_m)}{d\Delta_m} + \frac{\partial v^c}{d\Delta_m} \right\}
\]

\[
= (1 - p + p(1 - r + rG(v^c|I))) \times \left\{ \frac{\partial v^c dI(\Delta_m)}{d\Delta_m} + \frac{\partial v^c}{d\Delta_m} \right\}.
\]

Thus,

\[
\text{sgn} \frac{d}{d\Delta_m} W(v^c(I(\Delta_m), \Delta_m), I(\Delta_m))
\]

\[
= \text{sgn} \left( \frac{\partial v^c dI(\Delta_m)}{d\Delta_m} + \frac{\partial v^c}{d\Delta_m} \right)
\]

Now, recall from (46) that:

\[
\frac{dI(\Delta_m)}{d\Delta_m} = -\frac{prG_1(v^c|I) \times \frac{\partial v^c}{d\Delta_m}}{prG_1(v^c|I) \times \frac{\partial v^c}{d\Delta_m} + prG_{II}(v^c|I)v^c + pr \int v^c v_{gII}(v|I)dv},
\]

so

\[
\text{sgn} \left( \frac{\partial v^c dI(\Delta_m)}{d\Delta_m} + \frac{\partial v^c}{d\Delta_m} \right)
\]

\[
= \text{sgn} \left( \frac{\partial v^c}{d\Delta_m} \times \frac{-prG_1(v^c|I) \times \frac{\partial v^c}{d\Delta_m}}{prG_1(v^c|I) \times \frac{\partial v^c}{d\Delta_m} + prG_{II}(v^c|I)v^c + pr \int v^c v_{gII}(v|I)dv} + 1 \right)
\]

\[
= \text{sgn} \left( \frac{\partial v^c}{d\Delta_m} \times \frac{-H}{H + Q} + 1 \right),
\]

using the notation developed above in (44) and (45). Observe that \(\frac{-H}{H + Q} + 1 > 0\) is positive, since - as we previously noted - both \(H\) and \(Q\) are negative, and all expressions of the form \(\frac{-H}{H + Q}\) satisfy \(\frac{-H}{H + Q} > -1\), when both \(H\) and \(Q\) are negative. Since, by Theorem 1(a), \(\frac{\partial v^c}{d\Delta_m} < 0\) when \((1 - \tilde{f})\bar{\Delta}_m - \tilde{k} > 0\) and \(\frac{\partial v^c}{d\Delta_m} > 0\) when \((1 - \tilde{f})\bar{\Delta}_m - \tilde{k} < 0\), this completes the proof of the theorem.

In exactly the same fashion, we can show:

\[
\text{sgn} \frac{d}{d\gamma} W(v^c(I(\gamma), \gamma), I(\gamma)) = \text{sgn} \left( \frac{\partial v^c}{d\gamma} \right)
\]

for each \(\gamma \in \{\tilde{f}, \tilde{k}, \tilde{q}, p, r\}\).